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DISCUSSION PAPER

Bilateral Bargaining with Endogenous Status Quo

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Abstract We consider a non-cooperative bargaining game where in each round, if a proposal is rejected, with a probability, the allocation implemented in the previous round is implemented as a status quo and the game proceeds to the next round, and with the complementary probability, no allocation is implemented and the game terminates. We show that there uniquely exists a stationary subgame perfect equilibrium (SSPE) such that in any round, a proposer offers a proposal the acceptance and rejection of which are indifferent for a responder and an agreement is achieved. The SSPE allocations are convex combinations of the status quo and an allocation, and the sequence of SSPE allocations converges to this allocation as the period goes to infinity. The allocation converges to the equilibrium allocation of the Rubinstein bargaining game as the probability of the bargaining continuing after rejection goes to unity. The effect of the status quo on the SSPE allocations vinishes over time.

Keywords: Bargaining; Endogenous status quo; Breakdown; Relevance to the Rubinstein bargaining game; Vanishing effect of the status quo
JEL classification codes: C73; C78

1 Introduction

We consider a non-cooperative bilateral bargaining game in which the allocation implemented in the previous round becomes the status quo. In each round indexed by a status quo and a proposer determined by the events in the previous round, the player (proposer) proposes an allocation of a fixed surplus, and the other player (responder) accepts or rejects the proposal. If the responder accepts the proposal, it is implemented, and the game proceeds to the next round in which the implemented proposal becomes a status quo and the proposer remains a proposer. Otherwise, with a probability, the status quo is implemented, and the game proceeds to the next round in which the status quo remains a status quo and the responder becomes a proposer; with the complementary probability, no allocation is implemented, and the game terminates (breakdown).

We consider the existence of equiribria, the characterization of the equilibria, the relationship to the Rubinstein game and the effect of the status quos. We show that there uniquely exists a stationary subgame perfect equilibrium (SSPE) such that in any round, a proposer offers a proposal the acceptance and rejection of which are indifferent for a responder and an agreement is achieved. The SSPE allocations are convex combinations of the status quo and an allocation, and the sequence of SSPE allocations converges to this allocation as the period goes to infinity. The allocation converges to the equilibrium allocation of the Rubinstein bargaining game (Rubinstein (1982)) as the probability of the bargaining continuing after rejection goes to unity. The effect of the status quo on the SSPE allocations vinishes over time.

The legislative bargaining literature contains some investigations of endogenous status quos. In legislative bargaining, decisions are made by majority rule, whereas in our model, they are made by unanimity rule. Baron (1996) introduced endogenous status quos into legislative bargaining with a one-dimensional policy space and showed that the ideal point of the median player is asymptotically implemented. Kalandrakis (2004) and subsequent papers considered split-the-pie bargaining. Kalandrakis (2004) showed that a proposer eventually extracts the whole surplus in a three-player case.

Kalandrakis (2010) showed that an allocation exhibiting a minimal winning coalition is implemented in a five-or-more player case. Bowen and Zahran (2012) showed that a non-minimal winning coalition obtains positive payoffs under intermediate discount factors. Richter (2014) showed that when proposals can be non-exhaustive, i.e., the sum of payoffs in a proposal may be less than the whole surplus, egalitarian allocation is implemented. Anesi and Seidmann (2015) showed that when non-exhaustive proposals are allowed, a non-minimal winning coalition obtains positive payoffs.

Anesi and Seidmann (2015) also investigated bargaining under unanimity rule and showed that the equilibrium expected payoffs coincide with those in Baron and Ferejohn (1989). In their model, the initial status quo is a zero vector (non-exhaustive), and there is no risk of breakdown after rejection of a proposal, whereas in our model, the initial status quo is exhaustive, and there is a risk of breakdown. In both models, if the initial status quo were exhaustive and there were no risk of breakdown, trivially, the initial status quo would be implemented infinitely. In the equilibrium in Anesi and Seidmann (2015), where an exhaustive allocation is implemented in the first round and becomes a status quo in the next round, this allocation is constantly a status quo and implemented in all subsequent rounds, whereas in the equilibrium in our paper, the status quos and implemented allocations are infinitely updated.

Serrano (1997) formulated a bargaining game with another type of endogenous status quo, and showed that the core allocation is implemented in equilibrium. In their model, players simultaneously announce an allocation before bargaining and if all players announce the same allocation, this allocation becomes a status quo in the bargaining. In their model, the status quo is not updated during bargaining, whereas in our model, the status quo is allowed to be periodically updated, and thus, the dynamics of the status quos can be investigated.

The remainder of this paper is organized as follows. Section 2 describes the model. Section 3 presents the results. Section 4 discusses the remaining problems.

2 Model

We define an extensive form game G as follows. Let $N := \{1,2\}$, where N is the set of players. Let $X := \left\{x \in \mathbb{R}^N_+ \mid \sum_{i\in N} x_i = 1\right\}$, where X is the set of allocations of the fixed surplus totaling 1. Let $\bar{x}^0 \in X$, where $\bar{x}^0$ is the initial status quo, which is exogenously given. A *state* is an element in $X \times N$. There are infinite rounds classified by states. A round with state $(\bar{x}, i)$ is such that the status quo is allocation $\bar{x}$ and the proposer is player i. The game begins at a round with state $\left(\bar{x}^0, 1\right)$. In any round with state $(\bar{x}, i)$, bargaining proceeds as follows.

1. Player i proposes an allocation $x \in X$.
2. Player $j \neq i$ announces his/her acceptance or rejection of this proposal.

- If player j accepts it, proposal x is implemented and the game proceeds to a round with state (x, i).
- If player j rejects it,
  - with probability $\rho \in [0, 1)$, the status quo $\bar{x}$ is implemented and the game proceeds to a round with state $(\bar{x}, j)$.
  - with probability $1 - \rho$, the game ends.

Let $\delta \in [0, 1)$, where δ is the common discount factor. For any $i \in N$, if the game ends in period 1, player i's payoff is 0; if the game ends in some period $T = 2, 3, \ldots,$ player i's payoff is $\sum_{t=1}^{T-1} \delta^{t-1} x_i^t$, where x^t is the allocation implemented in each period $t = 1, \ldots, T-1$; if the game continues infinitely, player i's payoff is $\sum_{t=1}^{\infty} \delta^{t-1} x_i^t$, where x^t is the allocation implemented in each period $t = 1, 2, \ldots$.

Our equilibrium concept is a stationary subgame perfect equilibrium (SSPE). Stationarity is defined as follows.

Definition 1. A strategy tuple s is *stationary* if the proposals in s depend only on the states and the responses in s depend only on the states and offered proposals.

We focus on SSPEs that satisfy Properties 1 and 2 regarding strategy tuple s.

Property 1. In any round, acceptance and rejection of the proposal in s are indifferent for the responder given that both players behave according to s in the subsequent subgame.

Property 2. In any round, the proposal in s is accepted in s.

3 Results

Proposition 1 derives a unique SSPE that satisfies Properties 1 and 2, and Corollary 1 presents the sequence of SSPE allocations. For any $i \in N$, let x^{i*} be the element in X such that $x_i^{i*} = \frac{1}{1+\rho\delta}$ and $x_j^{i*} = \frac{\rho\delta}{1+\rho\delta}$, where $j \in N \setminus \{i\}$.

Proposition 1. *There uniquely exists an SSPE that satisfies Properties 1 and 2. Furthermore, it is such that in any round under any state* $(\bar{x}, i)$*, player* i *proposes* $\rho\bar{x} + (1-\rho)\, x^{i*}$ *and player* $j \neq i$ *accepts any proposal* y *if and only if* $y_j \geq \rho\bar{x}_j + (1-\rho)\, x_j^{i*}$.

Proof. See Appendix. □

Corollary 1. *In the unique SSPE that satisfies Properties 1 and 2, the allocation implemented in any period* t *is* $\rho^t\bar{x}^0 + \left(1-\rho^t\right) x^{1*}$ *and converges to* x^{1*} *as* t *goes to infinity.*

Proof. See the derivation of (1) in the proof of Proposition 1. □

Remark 1. As ρ goes to 1, x^{1*} converges to the equilibrium allocation in the Rubinstein bargaining game with a common discount factor (Rubinstein (1982)). Thus, if ρ is large, the SSPE allocations are approximately convex combinations of the status quo and the equilibrium allocation in the Rubinstein bargaining game, and the limit of their sequence as the period goes to infinity is approximately the equilibrium allocation in the Rubinstein bargaining game.

Remark 2. The effect of the status quo on the SSPE allocations vinishes over time. In other words, x^{1^*} is a steady allocation or an absorbing allocation.

In the proof of uniqueness, the derivation of the SSPE payoff tuple is an important step. Let s be an SSPE that satisfies Properties 1 and 2. Let $u^i(\bar{x})$ be the payoff tuple in s in any subgame starting from a proposal node under state $(\bar{x}, i)$, and $x^i(\bar{x})$ be the proposal under state $(\bar{x}, i)$. By Properties 1 and 2, for any $(\bar{x}, i) \in X \times N$ and any $j \in N \setminus \{i\}$,

$$\begin{aligned} x_j^i(\bar{x}) + \delta u_j^i\left(x^i(\bar{x})\right) &= \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right) \\ u_j^i(\bar{x}) &= x_j^i(\bar{x}) + \delta u_j^i\left(x^i(\bar{x})\right). \end{aligned}$$

By eliminating $x_j^i(\bar{x}) + \delta u_j^i\left(x^i(\bar{x})\right)$, the arguments in $\left(u_j^i\right)_{i,j \in N}$ become only a single state, and thus, this system of simultaneous functional equations is reduced to a system of simultaneous equations.

4 Discussion

This paper does not answer whether there exists an SSPE that does not satisfy Properties 1 or 2. To answer this question, solving the following system of simultaneous functional equations is an important step. Let s be an SSPE. Let $u^i(\bar{x})$ be the expected payoff tuple in s in any subgame starting from a proposal node under state $(\bar{x}, i)$. Then, if any responder accepts any proposal the acceptance and rejection of which are indifferent in s, $\left(u^i(\bar{x})\right)_{(\bar{x},i) \in X \times N}$ must satisfy the following system of simultaneous functional equations: for any $(\bar{x}, i) \in X \times N$ and any $j \in N \setminus \{i\}$,

$$\exists x^i \in \arg\max_{y^i \in X} \left(\mathbf{1}_{y_j^i + \delta u_j^i(y^i) \geq \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right)} \left(y_i^i + \delta u_i^i\left(y^i\right)\right) + \mathbf{1}_{y_j^i + \delta u_j^i(y^i) < \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right)} \rho\left(\bar{x}_i + \delta u_i^j(\bar{x})\right) \right)$$

$$u_i^i(\bar{x}) = \mathbf{1}_{x_j^i + \delta u_j^i(x^i) \geq \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right)} \left(x_i^i + \delta u_i^i\left(x^i\right)\right) + \mathbf{1}_{x_j^i + \delta u_j^i(x^i) < \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right)} \rho\left(\bar{x}_i + \delta u_i^j(\bar{x})\right)$$

$$u_j^i(\bar{x}) = \mathbf{1}_{x_j^i + \delta u_j^i(x^i) \geq \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right)} \left(x_j^i + \delta u_j^i\left(x^i\right)\right) + \mathbf{1}_{x_j^i + \delta u_j^i(x^i) < \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right)} \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right).$$

Appendix: Proof of Proposition 1

Lemmas Let s be a stationary strategy tuple. For any $(\bar{x}, i) \in X \times N$, let $u^i(\bar{x})$ be the payoff tuple in s in any subgame starting from a proposal node under state $(\bar{x}, i)$ and $x^i(\bar{x})$ be the proposal in s under state $(\bar{x}, i)$.

Lemma 1. *Suppose that s satisfies Property 2. Let $(\bar{x}, i) \in X \times N$. Then, $u^i_1(\bar{x}) + u^i_2(\bar{x}) = \frac{1}{1-\delta}$, and $\left(\bar{x}_1 + \delta u^i_1(\bar{x})\right) + \left(\bar{x}_2 + \delta u^i_2(\bar{x})\right) = \frac{1}{1-\delta}$.*

Proof. For any $t \in \mathbb{N}$, let x^t be the allocation implemented by s in period t of any subgame starting from a proposal node under state $(\bar{x}, i)$. Then, $u^i_1(\bar{x}) + u^i_2(\bar{x}) = \sum_{\tau\in\mathbb{N}} \delta^{\tau-1} x^\tau_1 + \sum_{\tau\in\mathbb{N}} \delta^{\tau-1} x^\tau_2 = \sum_{\tau\in\mathbb{N}} \delta^{\tau-1}\left(x^\tau_1 + x^\tau_2\right) = \sum_{\tau\in\mathbb{N}} \delta^{\tau-1} = \frac{1}{1-\delta}$. $\left(\bar{x}_1 + \delta u^i_1(\bar{x})\right) + \left(\bar{x}_2 + \delta u^i_2(\bar{x})\right) = (\bar{x}_1 + \bar{x}_2) + \delta\left(u^i_1(\bar{x}) + u^i_2(\bar{x})\right) = 1 + \delta\frac{1}{1-\delta} = \frac{1}{1-\delta}$. □

Lemma 2. *Suppose that s satisfies Properties 1 and 2. Also, suppose that for any $i \in N$ and any $\bar{x}, \bar{y} \in X$, $u^i_i(\bar{x}) < u^i_i(\bar{y})$ if $\bar{x}_i < \bar{y}_i$. Let $(\bar{x}, i) \in X \times N$ and $y \in X$. Then, $y_j + \delta u^i_j(y) \gtreqless \rho\left(\bar{x}_j + \delta u^j_j(\bar{x})\right)$ if and only if $y_j \gtreqless x^i_j(\bar{x})$.*

Proof. By the supposition on u^i_i and Lemma 1, $y_j \gtreqless x^i_j(\bar{x}) \Leftrightarrow y_i \lesseqgtr x^i_i(\bar{x}) \Leftrightarrow y_i + \delta u^i_i(y) \lesseqgtr x^i_i(\bar{x}) + \delta u^i_i\left(x^i(\bar{x})\right) \Leftrightarrow y_j + \delta u^i_j(y) \gtreqless x^i_j(\bar{x}) + \delta u^i_j\left(x^i(\bar{x})\right)$. Note that by Property 1, $x^i_j(\bar{x}) + \delta u^i_j\left(x^i(\bar{x})\right) = \rho\left(\bar{x}_j + \delta u^j_j(\bar{x})\right)$. Then, $y_j \gtreqless x^i_j(\bar{x}) \Leftrightarrow y_j + \delta u^i_j(y) \gtreqless \rho\left(\bar{x}_j + \delta u^j_j(\bar{x})\right)$. □

Existence For any $(\bar{x}, i) \in X \times N$, let $x^i(\bar{x}) := \rho\bar{x} + (1-\rho)\, x^{i*}$. Let s be a strategy tuple such that under any state $(\bar{x}, i)$, player i proposes $x^i(\bar{x})$ and player $j \neq i$ accepts any proposal y if and only if $y_j \geq x^i_j(\bar{x})$. Note that for any $i \in N$, $x^i_i(\bar{x}) = \rho\bar{x}_i + (1-\rho)\frac{1}{1+\rho\delta}$.

By the construction of s, s satisfies stationarity and Property 2.

For any $(\bar{x}, i) \in X \times N$ and any $t \in \mathbb{N}$, let $\hat{x}^{it}(\bar{x})$ be the allocation implemented by s in period t of any subgame starting from a proposal node under state $(\bar{x}, i)$. Let $(\bar{x}, i) \in X \times N$. By the mathematical induction, show that for any $t \in \mathbb{N}$,

$$\hat{x}^{it}_i(\bar{x}) = \rho^t \bar{x}_i + \left(1-\rho^t\right)\frac{1}{1+\rho\delta}. \tag{1}$$

$\hat{x}_i^{i1}(\bar{x}) = x_i^i(\bar{x}) = \rho^1 \bar{x}_i + \left(1-\rho^1\right)\frac{1}{1+\rho\delta}$. Let $t \in \mathbb{N}$. Suppose that $\hat{x}_i^{it}(\bar{x}) = \rho^t \bar{x}_i + \left(1-\rho^t\right)\frac{1}{1+\rho\delta}$. Then,

$$\begin{aligned}
\hat{x}_i^{it+1}(\bar{x}) &= x_i^i\left(\hat{x}^{it}(\bar{x})\right) = \rho \hat{x}_i^{it}(\bar{x}) + (1-\rho)\frac{1}{1+\rho\delta} \\
&= \rho\left(\rho^t \bar{x}_i + \left(1-\rho^t\right)\frac{1}{1+\rho\delta}\right) + (1-\rho)\frac{1}{1+\rho\delta} \\
&= \rho^{t+1}\bar{x}_i + \left(1-\rho^{t+1}\right)\frac{1}{1+\rho\delta}.
\end{aligned}$$

For any $(\bar{x}, i) \in X \times N$, let $u^i(\bar{x})$ be the payoff tuple in s in any subgame starting from a proposal node under state $(\bar{x}, i)$. Then, by (1), for any $(\bar{x}, i) \in X \times N$,

$$\begin{aligned}
u_i^i(\bar{x}) &= \sum_{t\in\mathbb{N}} \delta^{t-1}\hat{x}_i^{it}(\bar{x}) = \sum_{t\in\mathbb{N}} \delta^{t-1}\left(\rho^t \bar{x}_i + \left(1-\rho^t\right)\frac{1}{1+\rho\delta}\right) \\
&= \frac{\rho}{1-\rho\delta}\bar{x}_i + \frac{1}{(1-\delta)(1+\rho\delta)} - \frac{\rho}{(1-\rho\delta)(1+\rho\delta)} \\
&= \frac{1}{1-\rho\delta}\left(\rho\bar{x}_i + (1-\rho)\frac{1}{(1-\delta)(1+\rho\delta)}\right). \qquad (2)
\end{aligned}$$

Let $(\bar{x}, i) \in X \times N$. Let $j \in N \setminus \{i\}$. By Property 2, Lemma 1 and (2),

$$\begin{aligned}
x_j^i(\bar{x}) + \delta u_j^i\left(x^i(\bar{x})\right) &= u_j^i(\bar{x}) = \frac{1}{1-\delta} - u_i^i(\bar{x}) \\
&= \frac{1}{1-\delta} - \frac{1}{1-\rho\delta}\left(\rho\bar{x}_i + (1-\rho)\frac{1}{(1-\delta)(1+\rho\delta)}\right) \\
&= \frac{\rho}{1-\rho\delta}\left(\bar{x}_j + (1-\rho)\frac{\delta}{(1-\delta)(1+\rho\delta)}\right).
\end{aligned}$$

By (2),

$$\begin{aligned}
\rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right) &= \rho\left(\bar{x}_j + \delta\frac{1}{1-\rho\delta}\left(\rho\bar{x}_j + (1-\rho)\frac{1}{(1-\delta)(1+\rho\delta)}\right)\right) \\
&= \frac{\rho}{1-\rho\delta}\left(\bar{x}_j + (1-\rho)\frac{\delta}{(1-\delta)(1+\rho\delta)}\right).
\end{aligned}$$

Thus, $x_j^i(\bar{x}) + \delta u_j^i\left(x^i(\bar{x})\right) = \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right)$. Hence, s satisfies Property 1.

Let $(\bar{x}, i) \in X \times N$. Let $j \in N \setminus \{i\}$.

Consider player i's proposal node under state $(\bar{x}, i)$. His/her payoff in s at this node is $u_i^i(\bar{x})$. His/her payoff by one-shot deviation to any proposal y accepted in s is $y_i + \delta u_i^i(y)$. Because y is accepted in s, $y_j \geq x_j^i(\bar{x})$, i.e., $y_i \leq x_i^i(\bar{x})$. Thus, by (2), $y_i + \delta u_i^i(y) \leq x_i^i(\bar{x}) + \delta u_i^i(x^i(\bar{x}))$. Note that by Property 2, $x_i^i(\bar{x}) + \delta u_i^i(x^i(\bar{x})) = u_i^i(\bar{x})$. Then, $y_i + \delta u_i^i(y) \leq u_i^i(\bar{x})$. Player i's payoff by one-shot deviation to any proposal rejected in s is $\rho\left(\bar{x}_i + \delta u_i^j(\bar{x})\right)$. By Lemma 1, $\rho\left(\bar{x}_i + \delta u_i^j(\bar{x})\right) + \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right) = \frac{\rho}{1-\delta} \leq \frac{1}{1-\delta} = u_i^i(\bar{x}) + u_j^i(\bar{x})$. Note that by Properties 1 and 2, $\rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right) = x_j^i(\bar{x}) + \delta u_j^i(x^i(\bar{x})) = u_j^i(\bar{x})$. Then, $\rho\left(\bar{x}_i + \delta u_i^j(\bar{x})\right) \leq u_i^i(\bar{x})$. Thus, player i's proposal in s under state $(\bar{x}, i)$ is optimal.

Let $y \in X$. Consider player j's node to respond to y under state $(\bar{x}, i)$. His/her payoff by accepting and rejecting y at this node given the actions in s at the other nodes is $y_j + \delta u_j^i(y)$ and $\rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right)$, respectively. By Lemma 2, $y_j + \delta u_j^i(y) \geq \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right)$ if and only if $y_j \geq x_j^i(\bar{x})$. Thus, player j's responses in s under state $(\bar{x}, i)$ are optimal.

Thus, by the one-shot deviation principle, s is a subgame perfect equilibrium.

Hence, s is an SSPE that satisfies Properties 1 and 2.

Uniqueness Let s be an SSPE that satisfies Properties 1 and 2. For any $(\bar{x}, i) \in X \times N$, let $u^i(\bar{x})$ be the expected payoff tuple in s in any subgame starting from a proposal node under state $(\bar{x}, i)$ and $x^i(\bar{x})$ be the proposal in s under state $(\bar{x}, i)$.

Let $\bar{x} \in X$. Let $i, j \in N$ such that $i \neq j$. By Properties 1 and 2, $u_j^i(\bar{x}) = x_j^i(\bar{x}) + \delta u_j^i(x^i(\bar{x})) = \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right)$; by Lemma 1, $u_i^i(\bar{x}) + u_j^i(\bar{x}) = \frac{1}{1-\delta}$. Thus, $u_i^i(\bar{x}) + \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right) = \frac{1}{1-\delta}$. Hence, for any $i \in N$,

$$u_i^i(\bar{x}) = \frac{1}{1-\rho\delta}\left(\rho\bar{x}_i + (1-\rho)\frac{1}{(1-\delta)(1+\rho\delta)}\right). \tag{3}$$

Let $(\bar{x}, i) \in X \times N$. Let $j \in N \setminus \{i\}$.

By Property 2, $u_i^i(\bar{x}) = x_i^i(\bar{x}) + \delta u_i^i\left(x^i(\bar{x})\right)$. Thus, by (3),

$$\begin{aligned}&\frac{1}{1-\rho\delta}\left(\rho\bar{x}_i + (1-\rho)\frac{1}{(1-\delta)(1+\rho\delta)}\right)\\&= x_i^i(\bar{x}) + \delta\frac{1}{1-\rho\delta}\left(\rho x_i^i(\bar{x}) + (1-\rho)\frac{1}{(1-\delta)(1+\rho\delta)}\right).\end{aligned}$$

Hence, $x_i^i(\bar{x}) = \rho\bar{x}_i + (1-\rho)\frac{1}{1+\rho\delta}$. Therefore, $x^i(\bar{x}) = \rho\bar{x} + (1-\rho)x^{i*}$.

Let $y \in X$. Consider player j's node to respond to y under state $(\bar{x}, i)$. By Property 2, if $y_j = x_j^i(\bar{x})$, player j accepts y in s. Player j's payoff by accepting and rejecting y under state $(\bar{x}, i)$ given the actions in s at the other nodes is $y_j + \delta u_j^i(y)$ and $\rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right)$, respectively. By Lemma 2, $y_j + \delta u_j^i(y) \gtrless \rho\left(\bar{x}_j + \delta u_j^j(\bar{x})\right)$ if $y_j \gtrless x_j^i(\bar{x})$. Thus, if $y_j > x_j^i(\bar{x})$ and $y_j < x_j^i(\bar{x})$, player j accepts and rejects y in s, respectively. Hence, player j accepts y if and only if $y_j \geq x_j^i(\bar{x}) = \rho\bar{x}_j + (1-\rho)x_j^{i*}$. □

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