#0001

DISCUSSION PAPER

Partially Cooperative Games

Tomohiko Kawamori

December 2017; Revised in January 2018

FACULTY OF ECONOMICS
MEIJO UNIVERSITY

Partially Cooperative Games

Tomohiko Kawamori
Meijo University kawamori@meijo-u.ac.jp

January 22, 2018

Abstract In this paper, we present a general model in which mutually dependent negotiations are simultaneously conducted and define a solution concept for the model. We provide a sufficient condition for the solution to exist and show that the solutions approximately coincide with the equilibrium outcomes of extensive form games. We present a solution to the merger paradox as an application.

Keywords: Interdependent bargaining situations; Nash bargaining solutions; Existence of solutions; Noncooperative foundation; Merger paradox
JEL classification codes: C71; C72; C73; C78

1 Introduction

Interdependent bargaining situations are ubiquitous in the real world. For example, consider the following scenario:

> There are four firms, 1, 2, 3 and 4. Firms 1 and 2 form an R&D alliance and firms 3 and 4 also form an alliance. Each alliance simultaneously decides its level of R&D. Then, each firm independently decides its quantity or price.

Within each alliance, firms collectively decide their R&D level. It is usual to make collective decisions by bargaining. However, this decision making is more complex: in bargaining within each alliance, the profit frontier and disagreement point depend on the R&D level of the other alliance.[1]

In this paper, we present a general formalization of interdependent bargaining situations, a *partially cooperative game*. Each player belongs to some decision groups, each decision group chooses an alternative given a disagreement alternative, and each player's payoff is determined depending on the tuple of the alternatives chosen by the decision groups. Note that for any decision group, given an alternative tuple of the other decision groups, a bargaining problem within this decision group is naturally defined. We call an alternative tuple s^* a *Nash solution* in G if for any decision group C, the alternative of decision group C in s^* is the Nash bargaining solution to the bargaining problem within decision group C given the alternative tuple of the other decision groups in s^*. We also define a solution concept in the case where transfers are available within each decision group. We provide a sufficient condition for a Nash solution to exist. We show that Nash solutions with transfers approximately coincide with the equilibrium outcomes of extensive form games. We present a solution to the merger paradox as an application: two downstream firms respectively transact firm-specific intermediate goods with two upstream firms; by merger of the two downstream firms, the merged firm transacts two upstream firms, enjoys an advantageous bargaining po-

[1] For further examples in addition to the above scenario, some countries engage in a negotiation on a free trade agreement (FTA), and other some countries, another FTA; in a legislature, a committee decides on a matter, and another committee, another matter.

sition and obtains a large profit.

Our formulation includes some existing theoretical framework as special cases. In an extreme case where no player cooperates, a partially cooperative game and a Nash solution are equivalent to a strategic form game and a Nash equilibrium (Nash (1950b) and Nash (1951)), whereas in another extreme case where all players cooperate, they are equivalent to a bargaining problem and the Nash bargaining solution (Nash (1950a) and Nash (1953)). In intermediate cases, multiple players cooperate, but not all. Thus, we call G a *partially* cooperative game. If every pair of players cooperates, the other groups of players do not cooperate, and each pair is given two alternatives, a partially cooperative game and a Nash solution are equivalent to a network formation game and a pairwise stable network (Jackson and Wolinsky (1996)).

Rubinstein (1982) and Binmore et al. (1986) give the Nash bargaining solution a noncooperative foundation. Our paper gives any Nash solution a noncooperative foundation in a similar way. That is, we define extensive form games as follows: a player proposes an alternative, and the other players respond to the proposal by accepting or rejecting it; if all responders accept it, the proposal is implemented, and otherwise, the procedure is repeated. In our extensive form games, proposal-response procedures in multiple coalitions are simultaneously conducted.

Okada (2010) investigates the relationship between the Nash bargaining solution and stationary subgame perfect equilibria in coalitional bargaining games. In his extensive form games, the coalitions form endogenously, whereas in our extensive form games, they are exogenously given. In Okada (2010), only one negotiation is conducted in each period, whereas in our paper, multiple negotiations may be simultaneously conducted. This difference results in the difference in the uniqueness of the equilibrium outcome: in Okada (2010), the uniqueness holds, whereas in our paper, it may not hold.

Genicot and Ray (2006) investigate a situation where there are a principal and multiple agents, the principal bilaterally contracts with each agent, and each agent's payoff by her outside option is increasing in the number of uncontracted agents. Genicot and

Ray (2006) consider a noncooperative game such that once an agreement is achieved, the contract remains binding in subsequent periods. Genicot and Ray (2006) show that the principal simultaneously contracts with some agents and sequentially contracts with the other agents later; by doing so, she can exploit the agents. The situation (underlying environment) in Genicot and Ray (2006) is a special case of the situation of our paper. However, the noncooperative game (bargaining procedure) is different from that of our paper, in which an agreement is binding in one period. Thus, the results are different.

Bennett (1997) formulates interdependent bargaining situations, defines a solution, shows the existence of the solution and gives the solution a noncooperative foundation. In Bennett (1997), a player's payoff at the disagreement point within a coalition depends on only agreements in the other coalitions that she belongs to, whereas in our paper, it depends on agreements in the other coalitions that she does not belong to as well as belongs to. Moreover, in Bennett (1997), the set of attainable payoff tuples for a coalition does not depend on agreements in the other coalitions, whereas in our paper, it does depend.

Chakrabarti et al. (2011) present a game where some players cooperate and show existence of equilibria. In Chakrabarti et al. (2011), some players are noncooperative and the other players are cooperative among themselves; this situation is, by our paper's words, described as each noncooperative player forms a singleton decision group and cooperative players jointly form a single decisive group whose alternatives are the their action tuples. In Chakrabarti et al. (2011), disagreement alternatives are not introduced, and cooperative players are assumed to maximize the sum of their payoffs with respect to their action tuples, whereas in our paper, players in each decision group bargain over their alternatives.

In the literature, several papers study interdependent bargaining situations in particular circumstances: e.g., negotiations between an upstream firm (supplier) and a downstream firm (buyer) in Horn and Wolinsky (1988) and Chipty and Snyder (1999); negotiations between a firm and a union in Davidson (1988) and Zhao (1995); negotiations between a supplier of a public good and a consumer of it in Matsushima and Shinohara (2015).[2] The present paper provides them a unified viewpoint.

Horn and Wolinsky (1988) and Chipty and Snyder (1999) are related to our application on a solution to the merger paradox.

Horn and Wolinsky (1988) investigate the situation where upstream and downstream firms negotiate prices of intermediate goods, and subsequently downstream firms decide the quantities of final goods. They compare three cases: (i) two pairs of a upstream firm and a downstream firm simultaneously bargain; (ii) a single upstream firm bargains with two downstream firms respectively; (iii) a upstream firm and a downstream firm bargain. They show that the profitability of merger depends on whether the differentiated final goods are substitute or complementary. A result related to our application is that if the final goods are substitute, merger of downstream firms is not profitable. In Horn and Wolinsky (1988), only prices of intermediate goods are negotiated, and there are only two downstream firms before merger, whereas in our paper, both prices and quantities are negotiated, and there are three downstream firms before merger. These differences result in the difference in the profitability of merger between Horn and Wolinsky (1988) and our paper. Moreover, in Horn and Wolinsky (1988), since there are only two downstream firms, the merger paradox does not occur, whereas our paper presents an example in which merger is not profitable in market transactions (the merger paradox), but it is profitable in negotiated transactions.

Chipty and Snyder (1999) investigate the situation where there are one supplier and multiple buyers and the supplier and each buyer simultaneously negotiate a quantity and price. Chipty and Snyder (1999) provide a condition for the merger of two buyers to increase their profits. This increase is due to the enhancement of their bargaining position. In Chipty and Snyder (1999), buyers' products are perfectly heterogeneous, and thus, the merger paradox does not occur, which is the main difference

[2] Manea (2011) investigate bargaining in networks, where each pair of linked nodes (players) potentially produces a surplus. In his model, at each period, only one pair of players negotiates. Thus, his model is not closely related to interdependent bargaining situations.

from our paper.

The remainder of this paper is organized as follows: Section 2 defines partially cooperative games and Nash solutions, Section 3 provides a sufficient condition for a Nash solution to exist, Section 4 shows that Nash solutions approximately coincide with the equilibrium outcomes of extensive form games, Section 5 presents a solution to the merger paradox as an application, and Section 6 concludes the paper.

2 Notations and definitions

For any sets I and X, let X^I be the set of families of elements in X indexed by I. For readability, we denote the term of family x at index i by x^i as well as x_i. If the index is a coalition or a member of a coalition, we use subscripts; if the index is a discount factor, a period or a proposer, we use superscripts (a coalition, a discount factor et cetera are defined below).

2.1 Partially cooperative games

Definition 1. A *partially cooperative game* is a quadruple $\left(N, \mathcal{C}, ((S_C, \bar{s}_C))_{C\in\mathcal{C}}, (u_i)_{i\in N}\right)$ such that N is a nonempty finite set; $\mathcal{C}$ is a cover of N, i.e., a set of nonempty subsets of N such that $\bigcup \mathcal{C} = N$;[3] for any $C \in \mathcal{C}$, $(S_C, \bar{s}_C)$ is a pointed set, i.e., $\bar{s}_C \in S_C$; for any $i \in N$, $u_i : \prod_{C\in\mathcal{C}} S_C \to \mathbb{R}$.

N represents the set of players. $\mathcal{C}$ represents the set of decision groups, and each decision group makes a collective decision by bargaining. Note that $\mathcal{C}$ may not be a partition of N, and thus, a player may belong to multiple decision groups. For any decision group C, S_C represents the set of alternatives for decision group C and $\bar{s}_C$ represents the disagreement alternative of bargaining in decision group C. Any player i's preference over bargaining outcome tuples in $\prod_{C\in\mathcal{C}} S_C$ is represented by u_i.

Hereafter, fix a partially cooperative game $G = \left(N, \mathcal{C}, ((S_C, \bar{s}_C))_{C\in\mathcal{C}}, (u_i)_{i\in N}\right)$. Let $\boldsymbol{S} := \prod_{C\in\mathcal{C}} S_C$. For any $s \in \boldsymbol{S}$ and $C \in \mathcal{C}$, let s_{-C} be the restriction of s to $\mathcal{C} \setminus \{C\}$. For

[3] Some authors denote $\bigcup \mathcal{C}$ by $\bigcup_{C\in\mathcal{C}} C$.

any $C \in \mathcal{C}$, let $\boldsymbol{S}_{-C} := \{s_{-C} \mid s \in \boldsymbol{S}\}$. For any $s_C \in S_C$ and any $s_{-C} \in \boldsymbol{S}_{-C}$ for some $C \in \mathcal{C}$, let $(s_C, s_{-C}) \in \boldsymbol{S}$ such that $(s_C, s_{-C})_C = s_C$ and $(s_C, s_{-C})_{-C} = s_{-C}$.

2.2 Nash solutions

Definition 2. $s^* \in \boldsymbol{S}$ is a *Nash solution in* G if for any $C \in \mathcal{C}$, s^*_C is a solution to the maximization problem

$$\max_{s_C \in S_C} \prod_{i \in C} \left(u_i\left(s_C, s^*_{-C}\right) - u_i\left(\bar{s}_C, s^*_{-C}\right)\right)$$
$$\text{s.t.} \forall i \in C, u_i\left(s_C, s^*_{-C}\right) \geq u_i\left(\bar{s}_C, s^*_{-C}\right).$$

A Nash solution s^* can be interpreted as follows: any decision group $C \in \mathcal{C}$ establishes an expectation s^C_{-C} for the bargaining outcome tuple in the other decision groups and maximizes the Nash product for players in decision group C under the expectation; the expectations of all decision groups are rational, i.e., for any $C \in \mathcal{C}$, $s^C_{-C} = s^*_{-C}$.

2.3 Nash solutions with transfers

Let $(R_C)_{C \in \mathcal{C}}$ be such that for any $C \in \mathcal{C}$, $R_C = \left\{r \in \mathbb{R}^C \mid \sum_{i \in C} r_i = 0\right\}$. Let $\boldsymbol{R} := \prod_{C \in \mathcal{C}} R_C$.

Definition 3. $(s^*, r^*) \in \boldsymbol{S} \times \boldsymbol{R}$ is a *Nash solution with transfers in* G if for any $C \in \mathcal{C}$, $\left(s^*_C, r^*_C\right)$ is a solution to the maximization problem

$$\max_{(s_C, r_C) \in S_C \times R_C} \prod_{i \in C} \left(u_i\left(s_C, s^*_{-C}\right) + r_{Ci} - u_i\left(\bar{s}_C, s^*_{-C}\right)\right)$$
$$\text{s.t.} \forall i \in C, u_i\left(s_C, s^*_{-C}\right) + r_{Ci} \geq u_i\left(\bar{s}_C, s^*_{-C}\right).$$

A Nash solution with transfers in G is a Nash solution in the partially cooperative game such that the set of any decision group C's alternatives is $S_C \times R_C$, any decision group C's disagreement alternative is $(\bar{s}_C, \bar{r}_C)$, and any player's payoff is the sum of the value of u_i and transfers to her, where $\bar{r}_C \in R_C$ such that for any $i \in C$, $\bar{r}_{Ci} = 0$.

2.4 Examples

Example 1 demonstrates that strategic form games are included as special cases of partially cooperative games.

Example 1. Suppose that $\mathcal{C} = \{\{i\} \mid i \in N\}$. Then, the partially cooperative game G is regarded as the strategic form game $\hat{G} := \left(N, \left(\left(\hat{S}_i, u_i\right)\right)_{i \in N}\right)$, where for any $i \in N$, $\hat{S}_i := S_{\{i\}}$. Note that the definition of a Nash solution is reduced to

$$\forall i \in N, s^*_{\{i\}} \in \arg\max_{s_{\{i\}} \in S_{\{i\}}} \left(u_i\left(s_{\{i\}}, s^*_{-\{i\}}\right) - u_i\left(\bar{s}_{\{i\}}, s^*_{-\{i\}}\right)\right) = \arg\max_{s_{\{i\}} \in S_{\{i\}}} u_i\left(s_{\{i\}}, s^*_{-\{i\}}\right).$$

Then, for any $s^* \in \prod_{i \in N} S_{\{i\}}$, s^* is a Nash solution in G if and only if s^* is a Nash equilibrium in $\hat{G}$. Note that the Nash solution does not depend on the disagreement alternatives.

Example 2 demonstrates that bargaining problems are included as special cases of partially cooperative games.

Example 2. Suppose that $\mathcal{C} = \{N\}$, $\left\{(u_i(s_N))_{i \in N} \mid s_N \in S_N\right\}$ is compact and convex and there exists $s_N \in S_N$ such that for any $i \in N$, $u_i(s_N) > u_i(\bar{s}_N)$. Then, the partially cooperative game G is regarded as the bargaining problem $B := \left(F, (d_i)_{i \in N}\right)$, where $F := \left\{(u_i(s_N))_{i \in N} \mid s_N \in S_N\right\}$ and for any $i \in N$, $d_i := u_i(\bar{s}_N)$. Note that the definition of a Nash solution is reduced to

$$s^*_N \in \arg\max_{s_N \in S_N} \prod_{i \in N} \left(u_i(s_N) - u_i(\bar{s}_N)\right)$$
$$\text{s.t.} \forall i \in C, u_i(s_N) \geq u_i(\bar{s}_N).$$

Then, for any $s^*_N \in S_N$, s^*_N is a Nash solution in G if and only if $(u_i(s^*_N))_{i \in N}$ is a Nash bargaining solution in B.

Example 3 demonstrates that network formation models are included as special cases of partially cooperative games.

Example 3. Suppose that $\mathcal{C} = \left\{C \in 2^N \mid |C| = 2\right\}$, and for any $C \in \mathcal{C}$, $(S_C, \bar{s}_C) = (\{0,1\}, 0)$. Then, for any $\{i,j\} \in \mathcal{C}$, $s_{\{i,j\}} = 1$ ($s_{\{i,j\}} = 0$) is interpreted as i and j are linked (not linked); thus, an alternative tuple s is regarded as the graph $\left(N, \left\{\{i,j\} \in \mathcal{C} \mid s_{\{i,j\}} = 1\right\}\right)$, where N is the set of nodes and $\{C \in \mathcal{C} \mid s_C = 1\}$ is the set of edges. For any $s^* \in \boldsymbol{S}$, s^* is a Nash solution in G if and only if for any $\{i,j\} \in \mathcal{C}$,

(i) if $u_i\left(0, s^*_{-\{i,j\}}\right) > u_i\left(1, s^*_{-\{i,j\}}\right)$ or $u_j\left(0, s^*_{-\{i,j\}}\right) > u_j\left(1, s^*_{-\{i,j\}}\right)$, then $s^*_{\{i,j\}} = 0$, and

(ii) if $u_i\left(1, s^*_{-\{i,j\}}\right) > u_i\left(0, s^*_{-\{i,j\}}\right)$ and $u_j\left(1, s^*_{-\{i,j\}}\right) > u_j\left(0, s^*_{-\{i,j\}}\right)$, then $s^*_{\{i,j\}} = 1$,

i.e., by the contrapositions of (i) and (ii) and $\neg P \vee Q \Leftrightarrow P \to Q$,

(i) if $s^*_{\{i,j\}} = 1$, then $u_i(s^*) \geq u_i\left(0, s^*_{-\{i,j\}}\right)$ and $u_j(s^*) \geq u_j\left(0, s^*_{-\{i,j\}}\right)$, and

(ii) if $s^*_{\{i,j\}} = 0$, then $u_i(s^*) < u_i\left(1, s^*_{-\{i,j\}}\right)$ implies $u_j(s^*) \geq u_j\left(1, s^*_{-\{i,j\}}\right)$.

According to Jackson and Wolinsky (1996), for any $s^* \in \boldsymbol{S}$, s^* is pairwise stable if and only if for any $\{i,j\} \in \mathcal{C}$,

(i) if $s^*_{\{i,j\}} = 1$, then $u_i(s^*) \geq u_i\left(0, s^*_{-\{i,j\}}\right)$ and $u_j(s^*) \geq u_j\left(0, s^*_{-\{i,j\}}\right)$, and

(ii) if $s^*_{\{i,j\}} = 0$, then $u_i(s^*) < u_i\left(1, s^*_{-\{i,j\}}\right)$ implies $u_j(s^*) > u_j\left(1, s^*_{-\{i,j\}}\right)$.

Thus, being a Nash solution is slightly weaker than being pairwise stable.

3 Existence of a Nash solution

3.1 Existence of a Nash solution

Theorem 1 provides a sufficient condition for a Nash solution to exist.

Theorem 1. *Suppose that for any $C \in \mathcal{C}$, S_C is compact and convex in a Euclidean space. Suppose that for any $i \in N$, u_i is continuous, and for any $C \in \mathcal{C}$ with $C \ni i$ and any $s_{-C} \in \boldsymbol{S}_{-C}$, $u_i(\cdot, s_{-C})$ is concave. Then, there exists a Nash solution in G.*

Proof. See Appendix A. □

The sketch of the proof is as follows. Let $\tilde{G} := \left(\mathcal{C}, (S_C)_{C\in\mathcal{C}}, (\tilde{u}_C)_{C\in\mathcal{C}}\right)$ such that for any $C \in \mathcal{C}$, $\tilde{u}_C : \boldsymbol{S} \to \mathbb{R}$ such that for any $s \in \boldsymbol{S}$,

$$\tilde{u}_C(s) = \begin{cases} \prod_{i\in C}\left(u_i(s) - u_i(\bar{s}_C, s_{-C})\right) & \text{if } \forall i \in C\,(u_i(s) \geq u_i(\bar{s}_C, s_{-C})) \\ -1 & \text{otherwise.} \end{cases}$$

$\tilde{G}$ is a strategic form game. By the suppositions of this theorem, it is shown that for any $C \in \mathcal{C}$, S_C is nonempty, compact and convex, $\tilde{u}_C$ is upper semi-continuous, $\tilde{u}_C(\cdot, s_{-C})$ is quasi-concave, and $s_{-C} \mapsto \max_{s_C\in S_C} \tilde{u}_C(s_C, s_{-C})$ exists and is continuous. Thus, by the Corollary from Theorem 2 in Dasgupta and Maskin (1986), there exists a Nash equilibrium in $\tilde{G}$. Note that by the construction of $\tilde{G}$, the Nash equilibria in $\tilde{G}$ are equivalent to the Nash solutions in G.[4] Then, the conclusion is obtained.

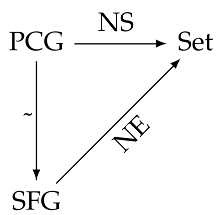
3.2 Existence of a Nash solution with transfers

Lemma 1 provides a necessary and sufficient condition for a pair to be a Nash solution with transfers. This lemma is useful to calculate Nash solutions with transfers.

Lemma 1. *For any* $(s^*, r^*) \in \boldsymbol{S} \times \boldsymbol{R}$, (s^*, r^*) *is a Nash solution with transfers in G if and only if for any* $C \in \mathcal{C}$, $s^*_C \in \arg\max_{s_C\in S_C} \sum_{i\in C} u_i\left(s_C, s^*_{-C}\right)$ *and for any* $i \in C$, $r^*_{Ci} = \frac{\sum_{j\in C}\left(u_j(s^*) - u_j\left(\bar{s}_C, s^*_{-C}\right)\right)}{|C|} - \left(u_i(s^*) - u_i\left(\bar{s}_C, s^*_{-C}\right)\right)$.

Proof. See Appendix B. □

[4]Let PCG be the class of partially cooperative games; SFG, the class of strategic form games; Set, the class of sets; NS, the map from PCG to Set such that each partially cooperative game maps to the set of its Nash solutions; NE, the map from SFG to Set such that each strategic form game maps to the set of its Nash equilibria. Then, the following diagram is commutative:



Theorem 2 provides a sufficient condition for a Nash solution with transfers to exist.

Theorem 2. *Suppose that for any $C \in \mathcal{C}$, S_C is compact and convex in a Euclidean space. Suppose that for any $i \in N$, u_i is continuous and for any $C \in \mathcal{C}$ with $C \ni i$ and any $s_{-C} \in \boldsymbol{S}_{-C}$, $u_i(\cdot, s_{-C})$ is concave. Then, there exists a Nash solution with transfers in G.*

Proof. See Appendix C. □

The sketch of the proof is as follows. Let $\tilde{G} := \left(\mathcal{C}, (S_C)_{C \in \mathcal{C}}, (\tilde{u}_C)_{C \in \mathcal{C}}\right)$ such that for any $C \in \mathcal{C}$, $\tilde{u}_C : \boldsymbol{S} \to \mathbb{R}$ such that for any $s \in \boldsymbol{S}$, $\tilde{u}_C(s) = \sum_{i \in C} u_i(s)$. $\tilde{G}$ is a strategic form game. By the suppositions of this theorem, it is shown that for any $C \in \mathcal{C}$, S_C is nonempty, compact and convex, $\tilde{u}_C$ is continuous, and $\tilde{u}_C(\cdot, s_{-C})$ is quasi-concave. Thus, by Theorem 1.2 in Fudenberg and Tirole (1991), there exists a Nash equilibrium s^* in $\tilde{G}$. Let $r^* \in \boldsymbol{R}$ such that for any $C \in \mathcal{C}$ and any $i \in C$, $r^*_{Ci} = \frac{\sum_{j \in C}\left(u_j(s^*) - u_j\left(\bar{s}_C, s^*_{-C}\right)\right)}{|C|} - \left(u_i(s^*) - u_i\left(\bar{s}_C, s^*_{-C}\right)\right)$. Then, by Lemma 1, (s^*, r^*) is a Nash solution with transfers in G. Then, the conclusion is obtained.

4 Noncooperative foundation

4.1 Extensive form games

Let Γ be the family indexed by $[0,1)$ such that for any discount factor $\delta \in [0,1)$, Γ^δ is the extensive form game defined as follows. The set of players is N. A *state* is $p \in N^{\mathcal{C}}$ such that for any $C \in \mathcal{C}$, $p_C \in C$. For any state p and any $C \in \mathcal{C}$, p_C represents the proposer in bargaining in C. There are infinite rounds, which are classified by states. In a round, there are $|\mathcal{C}|$ sessions, which are indexed by decision groups in $\mathcal{C}$. In a session with $C \in \mathcal{C}$ of a round with state p, player p_C proposes an element in $S_C \times R_C$, and then, each player in $C \setminus \{p_C\}$ sequentially announces whether she accepts or rejects the proposal, according to some predetermined total order $\precsim$ on N,[5] until a player rejects it or all players accept it. All sessions in a round are simultaneously conducted,

[5]The order of responses does not affect the results of this paper.

i.e., no player in a session knows the events in the other sessions. Let $\mathcal{A}$ be the set of $C \in \mathcal{C}$ such that in the session with C, all players in $C \setminus \{p_C\}$ accept the proposal. Then, for any $C \in \mathcal{A}$, the agreed-upon proposal in the session with C is implemented, and for any $C \in \mathcal{C} \setminus \mathcal{A}$, $\bar{s}_C$ with zero transfers is implemented. Then, the game proceeds to the next round with state p' such that for any $C \in \mathcal{A}$, $p'_C = p_C$ and for any $C \in \mathcal{C} \setminus \mathcal{A}$, p'_C is the rejecter in the session with C: a player whose proposal is accepted by all responders keeps confidence of the responders and thus retains a position as proposer, but a player whose proposal is rejected by a responder does not. The game starts with a round with some predetermined state. Proposers in the first round are specified by this initial state, and proposers in subsequent rounds are specified by states transiting as described above. Any player i's payoff in any complete history h is $\sum_{t \in \mathbb{N}} \delta^{t-1} \left(u_i\left(s^t\right) + \sum_{i \in C \in \mathcal{C}} r^t_{Ci}\right)$,[6] where for any $t \in \mathbb{N}$, $\left(s^t, r^t\right) \in \boldsymbol{S} \times \boldsymbol{R}$ such that for any $C \in \mathcal{C}$, $\left(s^t_C, r^t_C\right)$ is the allocation in the end of the session of the tth round in complete history h. Let Σ be the set of pairs of strategy tuple and belief system in Γ.

The information sets are as follows. Formally, on the game tree, the sessions within each round are arranged in some order. This order is arbitrary and has no substantial significance. For each player, her nodes n and n' are in the same information set if and only if the history reaching n is equivalent to the history reaching n' up to difference in the events in the sessions preceding n and n' of the round including n and n'. Consider a case where $\mathcal{C} = \{\{i,0\}, \{j,1\}\}$ with $i \neq 0$ and $j \neq 1$, Let the session with $\{i,0\}$ be the first session in each round. Consider a round with state $p_{\{i,0\}} = i$ and $p_{\{j,1\}} = j$. Let $n_j(x_i, y_0)$ be player j's node following player i's proposal x_i and player 0's response y_0, and $n_1\left(x_i, y_0, x'_j\right)$ be player 1's node following player i's proposal x_i, player 0's response y_0 and player j's proposal x'_j. Player j and 1 remember the events before the current round and the event in the current session, but they do not know the events in the session with $\{i,0\}$ of the current round. Thus, player j's information set is $\left\{n_j(x_i, y_0) \mid x_i \in S_{\{i,0\}} \times R_{\{i,0\}} \wedge y_0 \in \{a,r\}\right\} =: I_j$, and player 1's information sets are $\left\{n_1\left(x_i, y_0, x'_j\right) \mid x_i \in S_{\{i,0\}} \times R_{\{i,0\}} \wedge y_0 \in \{a,r\}\right\} =: I_1\left(x'_j\right)$ ($x'_j \in S_{\{j,1\}} \times R_{\{j,1\}}$),

[6] In this paper, $\mathbb{N} \not\ni 0$.

where a and r indicate acceptance and rejection, respectively.

This extensive form game is not necessarily a perfect-recall game. If $\mathcal{C}$ is not a partition of N, when some player involving multiple sessions is in a session, she does not know the events in the other sessions that she involves; thus, this extensive form game is generically an imperfect-recall game. If $\mathcal{C}$ is a partition of N, this extensive form game is a perfect-recall game. Consider the above-mentioned case and round. If $j = i$, $n_i(x_i, y_0)$s are in the same information set; thus, player i does not remember her proposals x_is offered to player 0; hence, the extensive form game is an imperfect-recall game. If $i \neq j$, the extensive form game is a perfect-recall game.

For any $\delta \in [0, 1)$, a *stationary perfect Bayesian equilibrium (SPBE) in* Γ^δ is a perfect Bayesian equilibrium such that any player's proposals depend only on states and any player's responses depend only on states, the first elements in the proposal and the transfers proposed to her. In the multilateral bargaining literature, it is common to refine equilibria by imposing stationarity. An *SPBE family in* Γ is $\sigma \in \Sigma^{[0,1)}$ such that for any $\delta \in [0, 1)$, σ^δ is an SPBE in Γ^δ. For any SPBE family σ in Γ, the *outcome of* σ is $(s, r) \in \left(\boldsymbol{S}^{\mathbb{N}}\right)^{[0,1)} \times \left(\boldsymbol{R}^{\mathbb{N}}\right)^{[0,1)}$ such that for any $\delta \in [0, 1)$, any $t \in \mathbb{N}$, and any $C \in \mathcal{C}$, $\left(s_C^{\delta t}, r_C^{\delta t}\right)$ is the allocation implemented in the session with C of the tth round in the complete history realized by σ^δ. For any SPBE family σ in Γ, σ is a *constant-alternative* SPBE family if for any $\delta, \delta' \in [0, 1)$, $s^\delta = s^{\delta'}$, where s is the first entry of the outcome of σ. For any SPBE family σ in Γ, σ is a *no-delay* SPBE family if for any $\delta \in [0, 1)$, any $C \in \mathcal{C}$ and any $i \in C$, player i's proposal by σ^δ in any session with C in which player i is the proposer is accepted by all responders in σ^δ.

Consistent belief systems are as follows. The belief system consistent with a strategy tuple σ is such that each player, in each information set I of her, assigns probability 1 to the node subsequent to the history induced by σ in the sessions preceding I of the minimum subgame including I. Consider the above-mentioned case and round. For example, consider a strategy tuple σ such that player i always proposes some $\hat{x}_i$ and player 0 always accepts proposal $\hat{x}_i$. Players j and 1's beliefs consistent with σ are such that player j assigns probability 1 to $n_j(\hat{x}_i, a)$ in I_j and for any $x'_j \in S_{\{j,1\}} \times R_{\{j,1\}}$, player 1 assigns provability 1 to $n_1\left(\hat{x}_i, a, x'_j\right)$ in $I_1\left(x'_j\right)$, respectively.

A stationary subgame perfect equilibrium (SSPE) cannot ensure sequential rationality. Suppose that $|\mathcal{C}| \geq 2$. Then, there is no subgame other than subgames starting from the initial node of a round. Thus, an SSPE may not satisfy sequential rationality of proposals and responses. Consider the above-mentioned case and round. For example, consider a strategy tuple such that every player always proposes the disagreement alternative and no transfer and every player always rejects proposals. This strategy tuple is an SSPE: in any subgame, by any deviation of any player, the disagreement alternative and no transfer are implemented in any session, and thus, her payoff does not improve. However, this strategy tuple is not sequentially rational: in $I_1\left(\bar{s}_{\{j,1\}}, r_{\{j,1\}}\right)$ with $r_{\{j,1\}1} > 1$, by one-shot deviation to acceptance, player 1's payoff improves by $r_{\{j,1\}1}$.

4.2 Equilibria

Definition 4. For any Nash solution (s^*, r^*) with transfers in G and any SPBE family σ in Γ, σ *supports* (s^*, r^*) if for any $t \in \mathbb{N}$, for any $\delta \in [0,1)$, $s^{\delta t} = s^*$ and $\lim_{\delta \to 1} r^{\delta t} = r^*$, where (s, r) is the outcome of σ.

Theorem 3 states that Nash solutions with transfers are supported by no-delay constant-alternative SPBE.

Theorem 3. *Suppose that $\mathcal{C}$ is a partition of N. (i) For any Nash solution (s^*, r^*) with transfers in G, for some no-delay constant-alternative SPBE family σ in Γ, σ supports (s^*, r^*). (ii) For any no-delay constant-alternative SPBE family σ in Γ, for some Nash solution (s^*, r^*) with transfers in G, σ supports (s^*, r^*).*

Proof. See Appendix D. □

The proof of (i) is constructive. Let (s^*, r^*) be a Nash solution with transfers in G. Let $\sigma \in \Sigma^{[0,1)}$ such that for any $\delta \in [0,1)$, in σ^δ, for any $i \in N$, in any session with $C \ni i$, (i) if player i is a proposer, player i proposes (s^*_C, r_C) such that for any

$j \in C \setminus \{i\}$, $r_{Cj} = \delta r^*_{Ci} - (1-\delta)\left(u_i(s^*) - u_i\left(\bar{s}_C, s^*_{-C}\right)\right)$; (ii) if player i is a responder, player i accepts a proposal (s_C, r_C) if and only if

$$u_i\left(s_C, s^*_{-C}\right) + r_{Ci} \geq \delta\left(u_i(s^*) + r^*_{Ci}\right) + (1-\delta)\, u_i\left(\bar{s}_C, s^*_{-C}\right).$$

Then, it is shown that σ is a no-delay constant-alternative SPBE family and supports (s^*, r^*).

4.3 The case where $\mathcal{C}$ may not be a partition

We give a noncooperative foundation to Nash solutions with transfers without the assumption that $\mathcal{C}$ is a partition.

We consider the situation where for any $(i, C) \in N \times \mathcal{C}$ such that $i \in C$, player i employs an agent, which is denoted by (i, C), in the sessions with C (e.g., if players are firms, each firm has employees in charge of decision groups), and agent (i, C)'s reward is strictly increasing in player i's payoff. These agents play extensive form games as in Subsection 4.1. Formally, we consider the following model. Define $\tilde{\Gamma}$ by the definition of Γ with the following replacements: "N" is replaced by "$\{(i, C) \in N \times \mathcal{C} \mid i \in C\}$"; "$p_C$," "$(p_C, C)$"; "$p'_C$," "$(p'_C, C)$"; "player i," "player (i, C)." Then, $\tilde{\Gamma}^\delta$ is a perfect-recall game. As in Γ, we define "stationary perfect Bayesian equilibrium (SPBE)," "SPBE family," "outcome," "constant-alternative" and "no-delay" in $\tilde{\Gamma}$.

That σ supports (s^*, r^*) in the case where $\mathcal{C}$ may not be a partition is defined similarly to Definition 4.

Theorem 4 is the counterpart of Theorem 3 in the case where $\mathcal{C}$ may not be a partition.

Theorem 4. *(i) For any Nash solution (s^*, r^*) with transfers in G, for some no-delay constant-alternative SPBE family σ in $\tilde{\Gamma}$, σ supports (s^*, r^*). (ii) For any no-delay constant-alternative SPBE family σ in $\tilde{\Gamma}$, for some Nash solution (s^*, r^*) with transfers in G, σ supports (s^*, r^*).*

Proof. It is shown similarly to the proof of Theorem 3. □

5 Application: A solution to the merger paradox

We present an application where merger is not profitable (i.e., a merged firm's profit is less than the sum of firms' profits before merger) in market transactions (the merger paradox), but it is profitable in negotiated transactions. This is a solution to the merger paradox. The key is that in negotiated transactions, the merged firm's bargaining position is enhanced.

5.1 Fundamentals

There are three upstream firms, 0, 1 and 2 and three downstream firms, $\{0\}$, $\{1\}$ and $\{2\}$. Let $N^U := \{0, 1, 2\}$ and $N^D := \{\{0\}, \{1\}, \{2\}\}$. Each upstream firm produces an intermediate good. Each upstream firm i's cost function is $c : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that for any $q \in \mathbb{R}_{\geq 0}$, $c(q) = 0$ (there is no cost for production). Each downstream firm can produce one unit of a final good from one unit of the intermediate good. For any $x \in \mathbb{R}$, let $|x|^+ := \max\{x, 0\}$. The inverse demand function of the final good is $p : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that for any $Q \in \mathbb{R}_{\geq 0}$, $p(Q) = |1 - Q|^+$.

We consider the effect of the merger of downstream firms $\{1\}$ and $\{2\}$. Let $\{1, 2\}$ denote the merged firm and $\hat{N}^D := \{\{0\}, \{1, 2\}\}$.

We consider the four scenarios: negotiated or market transactions before or after merger. Each scenario has pathological solutions or equilibria such that the price of the final good is zero. Thus, we focus on solutions or equilibria such that the price of the final good is positive.[7]

5.2 Negotiated transactions before merger

Each upstream firm i supplies the intermediate good to downstream firm $\{i\}$, and they decide the quantity and price of the intermediate good by bargaining. This situation is represented by the partially cooperative game, $G := \left(N, \mathcal{C}, ((S_C, \bar{s}_C))_{C \in \mathcal{C}}, (u_i)_{i \in N}\right)$, such that $N = N^U \cup N^D$; $\mathcal{C} = \left\{\{i, \{i\}\} \mid i \in N^U\right\}$; for any $C \in \mathcal{C}$, $S_C = \mathbb{R}_{\geq 0}$ and

[7] Also, if we employ a positive marginal cost, we can exclude such pathological solutions or equilibria with qualitative preservation of the desired results.

$\bar{s}_C = 0$; for any $i \in N^U$ and any $q \in \mathbb{R}^{\mathcal{C}}_{\geq 0}$, $u_i(q) = -c\left(q_{\{i,\{i\}\}}\right)$, and $u_{\{i\}}(q) = p\left(\sum_{C \in \mathcal{C}} q_C\right) q_{\{i,\{i\}\}}$.

Let (q^*, r^*) be a positive-price Nash solution with transfers in G. Then, by Lemma 1, for any $C \in \mathcal{C}$,

$$q^*_C \in \arg\max_{q_C \in \mathbb{R}_{\geq 0}} \left| 1 - \sum_{D \in \mathcal{C} \setminus \{C\}} q^*_D - q_C \right|^+ q_C = \left\{ \frac{1 - \sum_{D \in \mathcal{C} \setminus \{C\}} q^*_D}{2} \right\}$$
$$r^*_{Ci} = \frac{\left(1 - \sum_{D \in \mathcal{C}} q^*_D\right) q^*_C}{2},$$

where $i \in C \cap N^U$. Thus, for any $i \in N^U$, $q^*_{\{i,\{i\}\}} = \frac{1}{4}$, and $r^*_{\{i,\{i\}\}i} = \frac{1}{32}$. Conversely, by Lemma 1, it can be easily checked that such (q^*, r^*) is a positive-price Nash solution with transfers in G. Therefore, such (q^*, r^*) is the unique positive-price Nash solution with transfers in G.

5.3 Negotiated transactions after merger

Suppose that downstream firms $\{1\}$ and $\{2\}$ merge. The merged downstream firm $\{1,2\}$ buys the intermediate good from upstream firms 1 and 2 and bargains with them separately. This situation is represented by the partially cooperative game, $\hat{G} := \left(\hat{N}, \hat{\mathcal{C}}, \left((\hat{S}_C, \hat{\bar{s}}_C)\right)_{C \in \hat{\mathcal{C}}}, (\hat{u}_i)_{i \in \hat{N}}\right)$, such that $\hat{N} = N^U \cup \hat{N}^D$; $\hat{\mathcal{C}} = \left\{\{i, I\} \mid i \in N^U \wedge I \in \hat{N}^D \wedge i \in I\right\}$; for any $C \in \hat{\mathcal{C}}$, $\hat{S}_C = \mathbb{R}_{\geq 0}$ and $\hat{\bar{s}}_C = 0$; for any $(i, I) \in N^U \times \hat{N}^D$ with $i \in I$ and any $q \in \mathbb{R}^{\hat{\mathcal{C}}}_{\geq 0}$, $\hat{u}_i(q) = -c\left(q_{\{i,I\}}\right)$, and $\hat{u}_I(q) = p\left(\sum_{C \in \hat{\mathcal{C}}} q_C\right) \sum_{I \in C \in \hat{\mathcal{C}}} q_C$.

Let $(\hat{q}^*, \hat{r}^*)$ be a positive-price Nash solution with transfers in $\hat{G}$ that is *symmetric* in the sense that $\hat{q}^*_{\{1,\{1,2\}\}} = \hat{q}^*_{\{2,\{1,2\}\}}$. For any $i \in N^U$, let C_i be the element in $\hat{\mathcal{C}}$ such that $i \in C_i$. Then, by Lemma 1,

$$\begin{aligned}
\hat{q}^*_{C_0} &\in \arg\max_{q_{C_0} \in \mathbb{R}_{\geq 0}} \left|1 - \hat{q}^*_{C_1} - \hat{q}^*_{C_2} - q_{C_0}\right|^+ q_{C_0} = \left\{\frac{1 - \hat{q}^*_{C_1} - \hat{q}^*_{C_2}}{2}\right\} \\
\hat{q}^*_{C_1} &\in \arg\max_{q_{C_1} \in \mathbb{R}_{\geq 0}} \left|1 - \hat{q}^*_{C_0} - \hat{q}^*_{C_2} - q_{C_1}\right|^+ \left(\hat{q}^*_{C_2} + q_{C_1}\right) = \left\{\frac{\left|1 - \hat{q}^*_{C_0} - 2\hat{q}^*_{C_2}\right|^+}{2}\right\} \\
\hat{r}^*_{C_0 0} &= \frac{\left(1 - \hat{q}^*_{C_0} - \hat{q}^*_{C_1} - \hat{q}^*_{C_2}\right)\hat{q}^*_{C_0}}{2} \\
\hat{r}^*_{C_1 1} &= \frac{\left(1 - \hat{q}^*_{C_0} - \hat{q}^*_{C_1} - \hat{q}^*_{C_2}\right)\left(\hat{q}^*_{C_1} + \hat{q}^*_{C_2}\right) - \left(1 - \hat{q}^*_{C_0} - \hat{q}^*_{C_2}\right)\hat{q}^*_{C_2}}{2}.
\end{aligned}$$

Note that by the symmetry, $\frac{\left|1-\hat{q}^*_{C_0}-2\hat{q}^*_{C_2}\right|^+}{2} = \frac{\left|1-\hat{q}^*_{C_0}-\hat{q}^*_{C_1}-\hat{q}^*_{C_2}\right|^+}{2} = \frac{1-\hat{q}^*_{C_0}-\hat{q}^*_{C_1}-\hat{q}^*_{C_2}}{2}$. Then, for any $i \in \{1,2\}$, $\hat{q}^*_{\{0,\{0\}\}} = \frac{1}{3}$, $\hat{q}^*_{\{i,\{1,2\}\}} = \frac{1}{6}$, $\hat{r}^*_{\{0,\{0\}\}0} = \frac{1}{18}$ and $\hat{r}^*_{\{i,\{1,2\}\}i} = \frac{1}{72}$. Conversely, by Lemma 1, it can be easily checked that such $(\hat{q}^*, \hat{r}^*)$ is a symmetric positive-price Nash solution with transfers in $\hat{G}$. Therefore, such $(\hat{q}^*, \hat{r}^*)$ is the unique symmetric positive-price Nash solution with transfers in $\hat{G}$.

5.4 Market transactions before merger

Each upstream firm independently decides the price of its intermediate good, and then, each downstream firm independently decides the quantity of its final good. Each downstream firm buys the intermediate good from a upstream firm with the lowest price. Let $\daleth$ denote this extensive form game.

Let $(q^\circ, w^\circ) \in \mathbb{R}^{N^D}_{\geq 0} \times \mathbb{R}_{\geq 0}$ be an equilibrium outcome in $\daleth$, i.e., the tuple of the downstream firms' quantities and the lowest price of the intermediate good in the play of some positive-price subgame perfect equilibrium (SPE) in $\daleth$. Then, for the same reason as in the Bertrand game, $w^\circ = 0$. Thus, for any $I \in N^D$,

$$q^\circ_I \in \arg\max_{q_I \in \mathbb{R}_{\geq 0}} \left|1 - \sum_{J \in N^D \setminus \{I\}} q^\circ_J - q_I\right|^+ q_I = \left\{\frac{1 - \sum_{J \in N^D \setminus \{I\}} q^\circ_J}{2}\right\}.$$

Hence, for any $I \in N^D$, $q^\circ_I = \frac{1}{4}$. It can be easily checked that there exists a positive-price SPE in $\daleth$. Therefore, such (q°, w°) is the unique equilibrium outcome in $\daleth$.

5.5 Market transactions after merger

Suppose that downstream firms $\{1\}$ and $\{2\}$ merge. Each upstream firm independently decides the price of its intermediate good, and then, each downstream firm independently decides the quantity of its final good. Each downstream firm buys the intermediate good from a upstream firm with the lowest price. Let $\hat{\daleth}$ denote this extensive form game.

Similarly to $\daleth$, $(\hat{q}^\circ, \hat{w}^\circ) \in \mathbb{R}_{\geq 0}^{\hat{N}^D} \times \mathbb{R}_{\geq 0}$ such that for any $I \in \hat{N}^D$, $\hat{q}_I^\circ = \frac{1}{3}$ and $\hat{w}^\circ = 0$ is the unique equilibrium outcome in $\hat{\daleth}$, i.e., the tuple of the downstream firms' quantities and the lowest price of the intermediate good in the play of some positive-price SPE in $\hat{\daleth}$.

5.6 Comparison

Let Δ^* be the change of the profit of firms $\{1\}$ and $\{2\}$ by merger in negotiated transactions and Δ° be that in market transactions. Then,

$$\Delta^* = \left(\left(1 - \frac{1}{3} - 2 \cdot \frac{1}{6}\right) \cdot 2 \cdot \frac{1}{6} - 2 \cdot \frac{1}{72}\right) - 2\left(\left(1 - 3 \cdot \frac{1}{4}\right)\frac{1}{4} - \frac{1}{32}\right) = \frac{1}{48} > 0$$
$$\Delta^\circ = \left(\left(1 - 2 \cdot \frac{1}{3}\right)\frac{1}{3} - 0 \cdot \frac{1}{3}\right) - 2\left(\left(1 - 3 \cdot \frac{1}{4}\right) \cdot \frac{1}{4} - 0 \cdot \frac{1}{4}\right) = -\frac{1}{72} < 0.$$

Since $\Delta^\circ < 0$, in the market transactions, the merger reduces the profit of firms $\{1\}$ and $\{2\}$. This is a sort of merger paradox. However, since $\Delta^* > 0$, in the negotiated transactions, the merger improves the profit of firms $\{1\}$ and $\{2\}$.

Improvement of the merged firm's profit in negotiated transactions is because the merged firm is in an advantageous bargaining position as explained as follows. When the merged firm and upstream firm 1 disagree, the quantity via firm 1's intermediate good is zero, but the quantity via upstream firm 2's one is positive. Since the quantity via firm 1's intermediate good is zero, the aggregate quantity is small, and thus, the price of the final good is high. Hence, the merged firm's disagreement payoff is high.

Therefore, the merged firm's bargaining position is strong. The detail is explained below.

$2\Delta^*$ is decomposed as

$$\begin{aligned}
&\sum_{C\in\hat{\mathcal{C}}\setminus\{\{0,\{0\}\}\}} p\left(\hat{q}^*_{\{0,\{0\}\}}+\hat{q}^*_C\right)\hat{q}^*_C - p\left(\sum_{C\in\mathcal{C}} q^*_C\right)\sum_{C\in\mathcal{C}\setminus\{\{0,\{0\}\}\}} q^*_C \\
&= \left(p\left(\sum_{C\in\mathcal{C}}\hat{q}^*_C\right)\sum_{C\in\hat{\mathcal{C}}\setminus\{\{0,\{0\}\}\}}\hat{q}^*_C - p\left(\sum_{C\in\mathcal{C}} q^*_C\right)\sum_{C\in\hat{\mathcal{C}}\setminus\{\{0,\{0\}\}\}} q^*_C\right) \\
&\quad + \sum_{C\in\hat{\mathcal{C}}\setminus\{\{0,\{0\}\}\}}\left(p\left(\hat{q}^*_{\{0,\{0\}\}}+\hat{q}^*_C\right) - p\left(\sum_{D\in\mathcal{C}}\hat{q}^*_D\right)\right)\hat{q}^*_C.
\end{aligned}$$

The first term of the right hand side is the effect on the revenue: by the merger, the quantity tuple changes from q^* to $\hat{q}^*$; thus, the first term is the changes of the revenue of the merged firm. Since $q^*_{\{i,\{i\}\}} = q^\circ_{\{i\}}$ for any $i \in N^U$, $\hat{q}^*_{\{0,\{0\}\}} = \hat{q}^\circ_{\{0\}}$ and $\sum_{C\in\hat{\mathcal{C}}\setminus\{\{0,\{0\}\}\}}\hat{q}^*_C = q^\circ_{\{1,2\}}$, it is equal to Δ°, which is negative (the merger paradox).

The second term of the right hand side is the effect on the transfers: by the merger, upstream firm 1's counterpart changes from $\{1\}$ and $\{1,2\}$; thus, $p\left(\sum_{D\in\hat{\mathcal{C}}}\hat{q}^*_D\right)\hat{q}^*_{\{2,\{1,2\}\}}$ and $p\left(\hat{q}^*_{\{0,\{0\}\}}+\hat{q}^*_{\{2,\{1,2\}\}}\right)\hat{q}^*_{\{2,\{1,2\}\}}$ are added to the counterpart's payoffs in agreement and disagreement, respectively; hence, the difference between these payoffs is equal to the change of the surplus in bargaining associated with upstream firm 1 and proportional to the change of the transfer to upstream firm 1; the sum of this difference and its analog to upstream firm 2 is the second term above. Since the inverse demand function is decreasing, the second term is positive. This can be interpreted as the counterpart's added payoff in disagreement is greater than that in agreement, and thus, her bargaining position is enhanced by the merger.

Under the inverse demand function considered here, the second term dominates the first term, and thus, the merged firm's profit increases by the merger.

The standard solution to the merger paradox is that the *synergy effect* reduces production costs and makes merger profitable. In this application, cost synergy is absent, but the merger is profitable. Therefore, this application gives the merger paradox another solution, which is led to by the *bargaining effect*.

6 Conclusion

We conclude the paper by presenting several open questions. (i) Our existence results were proven through the theorems of existence of Nash equilibria based on Kakutani's fixed point theorem. Meanwhile, there are other theorems on existence of Nash equilibria (e.g., the theorem of Topkis (1979), which is based on Tarski's fixed point theorem. How is the results on existence of Nash solutions proven through other theorems of existence of Nash equilibria. (ii) Under the protocol of our extensive form game, if a proposal is rejected by some responder, the rejecter becomes a proposer in the current round; otherwise, the proposer in the previous round continues to be a proposer in the current round. Meanwhile, some studies on noncooperative bargaining use other protocols (e.g., the random-proposer protocol in Okada (1996)). Is our noncoopearative foundation result robust to protocols? (iii) We gave no noncooperative foundation to Nash solutions *without transfers*. What is noncooperative foundation for Nash solutions without transfers? (iv) While the Nash bargaining solution is unique, Nash equilibria are not necessarily unique. When is a Nash solution unique? (v) While the Nash bargaining solution is Pareto efficient, Nash equilibria are not necessarily Pareto efficient. When are Nash solutions Pareto efficient?

Appendix

A Proof of Theorem 1

Let $\tilde{G} := \left(\mathcal{C}, (S_C)_{C\in\mathcal{C}}, (\tilde{u}_C)_{C\in\mathcal{C}}\right)$, where for any $C \in \mathcal{C}$, $\tilde{u}_C : \boldsymbol{S} \to \mathbb{R}$ such that for any $s \in \boldsymbol{S}$,

$$\tilde{u}_C(s) = \begin{cases} \prod_{i\in C}\left(u_i(s) - u_i(\bar{s}_C, s_{-C})\right) & \text{if } \forall i \in C\,(u_i(s) \geq u_i(\bar{s}_C, s_{-C})) \\ -1 & \text{otherwise.} \end{cases}$$

Note that $\tilde{G}$ is a strategic form game. By the definition of $\tilde{G}$, for any $s \in \boldsymbol{S}$, s is a Nash solution in partially cooperative game G if and only if s is a Nash equilibrium in strategic form game $\tilde{G}$. Thus, it suffices to show that there exists a Nash equilibrium in $\tilde{G}$.

Let $C \in \mathcal{C}$.

(i) S_C is nonempty, compact and convex.

(ii) Let $\boldsymbol{S}^- := \boldsymbol{S} \setminus \{s \in \boldsymbol{S} \mid \forall i \in C\,(u_i(s) \geq u_i(\bar{s}_C, s_{-C}))\}$. For any $s \in \boldsymbol{S}$, $\tilde{u}_C(s) = \prod_{i\in C}\max\{u_i(s) - u_i(\bar{s}_C, s_{-C}), 0\} - \chi_{\boldsymbol{S}^-}(s)$, where $\chi_{\boldsymbol{S}^-}$ is the characteristic function of subset $\boldsymbol{S}^-$ of $\boldsymbol{S}$, i.e., $\chi_{\boldsymbol{S}^-} : \boldsymbol{S} \to \{0, 1\}$ such that for any $s \in \boldsymbol{S}$, $\chi_{\boldsymbol{S}^-}(s) = 1$ if and only if $s \in \boldsymbol{S}^-$. Since for any $i \in C$, u_i is continuous, $s \mapsto \prod_{i\in C}\max\{u_i(s) - u_i(\bar{s}_C, s_{-C}), 0\}$ is continuous; $\boldsymbol{S}^-$ is open; thus, $\chi_{\boldsymbol{S}^-}$ is lower semi-continuous. Then, $\tilde{u}_C$ is upper semi-continuous.

(iii) Let $s_{-C} \in \boldsymbol{S}_{-C}$. Let $S_C^+ := \{s_C \in S_C \mid \forall i \in C\,(u_i(s) \geq u_i(\bar{s}_C, s_{-C}))\}$. Let $s_C, t_C \in S_C$ and $\lambda \in [0, 1]$. If $s_C \notin S_C^+$ or $t_C \notin S_C^+$, $\tilde{u}_C(s_C, s_{-C}) = -1$ or $\tilde{u}_C(t_C, s_{-C}) = -1$; thus,

$$\tilde{u}_C(\lambda s_C + (1-\lambda)t_C, s_{-C}) \geq -1 = \min\{\tilde{u}_C(s_C, s_{-C}), \tilde{u}_C(t_C, s_{-C})\}.$$

Suppose that $s_C \in S_C^+$ and $t_C \in S_C^+$. Since for any $i \in C$, $u_i(\cdot, s_{-C})$ is concave, S_C^+ is convex. For any $i \in C$, since $u_i(\cdot, s_{-C})$ is concave, $u_i(\cdot, s_{-C}) - u_i(\bar{s}_C, s_{-C})$ is concave; thus, the restriction of $u_i(\cdot, s_{-C}) - u_i(\bar{s}_C, s_{-C})$ to S_C^+ is concave; it is non-negative;

hence, it is logarithmically concave. Note that the product of logarithmically concave functions is logarithmically concave and the product of the restrictions of functions to a set is the restriction of the product of these functions to this set. Then, the restriction of $\tilde{u}_C(\cdot, s_{-C})$ to S_C^+ is logarithmically concave, thus, it is quasi-concave. Hence, since $s_C, t_C, \lambda s_C + (1-\lambda) t_C \in S_C$,

$$\tilde{u}_C(\lambda s_C + (1-\lambda) t_C, s_{-C}) \geq \min\{\tilde{u}_C(s_C, s_{-C}), \tilde{u}_C(t_C, s_{-C})\}.$$

Thus, $\tilde{u}_C(\cdot, s_{-C})$ is quasi-concave.

(iv) Let $\hat{u}_C : \boldsymbol{S} \to \mathbb{R}$ such that for any $s \in \boldsymbol{S}$, $\hat{u}_C(s) = \prod_{i \in C} \max\{u_i(s) - u_i(\bar{s}_C, s_{-C}), 0\}$. Since for any $i \in C$, u_i is continuous, $\hat{u}_C$ is continuous. Let $s_C^* : \boldsymbol{S}_{-C} \to S_C$ such that for any $s_{-C} \in \boldsymbol{S}_{-C}$, $s_C^*(s_{-C}) \in \arg\max_{s_C \in S_C} \hat{u}_C(s_C, s_{-C})$ and if $\max_{s_C \in S_C} \hat{u}_C(s_C, s_{-C}) = 0$, $s_C^*(s_{-C}) = \bar{s}_C$. Let $\hat{u}_C^* : \boldsymbol{S}_{-C} \to \mathbb{R}$ such that for any $s_{-C} \in \boldsymbol{S}_{-C}$, $\hat{u}_C^*(s_{-C}) = \max_{s_C \in S_C} \hat{u}_C(s_C, s_{-C})$. Note that since S_C is compact and $\hat{u}_C$ is continuous, by the Berge maximum theorem and the axiom of choice, there exist such s_C^* and $\hat{u}_C^*$, and $\hat{u}_C^*$ is continuous. Let $\tilde{u}_C^* : \boldsymbol{S}_{-C} \to \mathbb{R}$ such that for any $s_{-C} \in \boldsymbol{S}_{-C}$, $\tilde{u}_C^*(s_{-C}) = \max_{s_C} \tilde{u}_C(s_C, s_{-C})$. Note that since S_C is compact and $\tilde{u}_C$ is upper semi-continuous, by the Weierstrass extremum theorem, there exists such $\tilde{u}_C^*$. Let $s_{-C} \in \boldsymbol{S}_{-C}$. Since by the definition of s_C^*, for any $i \in C$, $u_i\left(s_C^*(s_{-C}), s_{-C}\right) \geq u_i(\bar{s}_C, s_{-C})$, $\tilde{u}_C(s^*(s_{-C}), s_{-C}) = \hat{u}_C(s^*(s_{-C}), s_{-C})$. Thus, by the definition of s_C^*, $\hat{u}_C$ and $\tilde{u}_C$, for any $s_C \in S_C$, $\tilde{u}_C\left(s_C^*(s_{-C}), s_{-C}\right) = \hat{u}_C\left(s_C^*(s_{-C}), s_{-C}\right) \geq \hat{u}_C(s_C, s_{-C}) \geq \tilde{u}_C(s_C, s_{-C})$. Hence, $s_C^*(s_{-C}) \in \arg\max_{s_C \in S_C} \tilde{u}_C(s_C, s_{-C})$. Therefore, $\tilde{u}_C^*(s_{-C}) = \tilde{u}_C(s^*(s_{-C}), s_{-C}) = \hat{u}_C(s^*(s_{-C}), s_{-C}) = \hat{u}_C^*(s_{-C})$. Thus, since $\hat{u}_C^*$ is continuous, $\tilde{u}_C^*$ is continuous.

From (i)–(iv) for each $C \in \mathcal{C}$, by the Corollary from Theorem 2 in Dasgupta and Maskin (1986), there exists a Nash equilibrium in $\tilde{G}$. □

B Proof of Lemma 1

Let $C \in \mathcal{C}$. Let $s_{-C} \in \boldsymbol{S}_{-C}$. For any $i \in C$, let $\tilde{u}_i : S_C \to \mathbb{R}$ such that for any $s_C \in S_C$, $\tilde{u}_i(s_C) = u_i(s_C, s_{-C})$. For any $s_C \in S_C$ such that $\sum_{i \in C}(\tilde{u}_i(s_C) - \tilde{u}_i(\bar{s}_C)) < 0$, there exists no $r_C \in R_C$ such that for any $i \in C$, $\tilde{u}_i(s_C) + r_{Ci} \geq \tilde{u}_i(\bar{s}_C)$. Let $s_C \in S_C$ such that $\sum_{i \in C}(\tilde{u}_i(s_C) - \tilde{u}_i(\bar{s}_C)) \geq 0$. Let $r_C^\circ \in R_C$ such that for any $i \in C$, $r_{Ci}^\circ = \frac{\sum_{j \in C}\left(\tilde{u}_j(s_C) - \tilde{u}_j(\bar{s}_C)\right)}{|C|} - (\tilde{u}_i(s_C) - \tilde{u}_i(\bar{s}_C))$. Note that for any $i \in C$, $\tilde{u}_i(s_C) + r_{Ci}^\circ = \frac{\sum_{j \in C}\left(\tilde{u}_j(s_C) - \tilde{u}_j(\bar{s}_C)\right)}{|C|} + \tilde{u}_i(\bar{s}_C) \geq \tilde{u}_i(\bar{s}_C)$. Let $r_C \in R_C \setminus \{r_C^\circ\}$ such that for any $i \in C$, $\tilde{u}_i(s_C) + r_{Ci} \geq \tilde{u}_i(\bar{s}_C)$. For any $i, j \in C$, $\tilde{u}_i(s_C) + r_{Ci}^\circ - \tilde{u}_i(\bar{s}_C) = \tilde{u}_j(s_C) + r_{Cj}^\circ - \tilde{u}_j(\bar{s}_C)$. Since $r_C \neq r_C^\circ$ and $r_C, r_C^\circ \in R_C$, for some $i, j \in C$, $\tilde{u}_i(s_C) + r_{Ci} - \tilde{u}_i(\bar{s}_C) \neq \tilde{u}_j(s_C) + r_{Cj} - \tilde{u}_j(\bar{s}_C)$. Thus, by the inequality of arithmetic and geometric means,

$$\begin{aligned}
\sqrt[|C|]{\prod_{i \in C}(\tilde{u}_i(s_C) + r_{Ci} - \tilde{u}_i(\bar{s}_C))} &< \frac{\sum_{i \in C}(\tilde{u}_i(s_C) + r_{Ci} - \tilde{u}_i(\bar{s}_C))}{|C|} \\
&= \frac{\sum_{i \in C}\left(\tilde{u}_i(s_C) + r_{Ci}^\circ - \tilde{u}_i(\bar{s}_C)\right)}{|C|} \\
&= \sqrt[|C|]{\prod_{i \in C}\left(\tilde{u}_i(s_C) + r_{Ci}^\circ - \tilde{u}_i(\bar{s}_C)\right)}.
\end{aligned}$$

Thus, the unique maximizer and maximum of

$$\begin{aligned}
&\max_{r_C \in R_C} \prod_{i \in C}(\tilde{u}_i(s_C) + r_{Ci} - \tilde{u}_i(\bar{s}_C)) \\
&\text{s.t.} \forall i \in C, \tilde{u}_i(s_C) + r_{Ci} \geq \tilde{u}_i(\bar{s}_C),
\end{aligned}$$

are r_C° and

$$\prod_{i \in C}(\tilde{u}_i(s_C) + r_{Ci}^\circ - \tilde{u}_i(\bar{s}_C)) = \left(\frac{\sum_{i \in C}\tilde{u}_i(s_C) - \sum_{i \in C}\tilde{u}_i(\bar{s}_C)}{|C|}\right)^{|C|},$$

respectively. Hence, the set of maximizers of this maximum with respect to $s_C \in S_C$ such that $\sum_{j \in C}\left(\tilde{u}_j(s_C) - \tilde{u}_{Ci}(\bar{s}_C)\right) \geq 0$ is equal to $\arg\max_{s_C \in S_C} \sum_{i \in C}\tilde{u}_i(s_C)$. Thus, for any $\left(s_C^*, r_C^*\right) \in S_C \times R_C$, $\left(s_C^*, r_C^*\right)$ is a maximizer of

$$\begin{aligned}
&\max_{(s_C, r_C) \in S_C \times R_C} \prod_{i \in C}(\tilde{u}_i(s_C) + r_{Ci} - \tilde{u}_i(\bar{s}_C)) \\
&\text{s.t.} \forall i \in C, \tilde{u}_i(s_C) + r_{Ci} \geq \tilde{u}_i(\bar{s}_C)
\end{aligned}$$

if and only if $s_C^* \in \arg\max_{s_C \in S_C} \sum_{i \in C} \tilde{u}_i(s_C)$ and for any $i \in C$, $r_{Ci} = \frac{\sum_{j \in C}\left(\tilde{u}_j\left(s_C^*\right) - \tilde{u}_j(\bar{s}_C)\right)}{|C|} - \left(\tilde{u}_i\left(s_C^*\right) - \tilde{u}_i(\bar{s}_C)\right)$. Therefore, the conclusion of this lemma is obtained. □

C Proof of Theorem 2

Let $\tilde{G} := \left(\mathcal{C}, (S_C)_{C \in \mathcal{C}}, (\tilde{u}_C)_{C \in \mathcal{C}}\right)$, where for any $C \in \mathcal{C}$, $\tilde{u}_C : \boldsymbol{S} \to \mathbb{R}$ such that for any $s \in \boldsymbol{S}$, $\tilde{u}_C(s) = \sum_{i \in C} u_i(s)$. Note that $\tilde{G}$ is a strategic form game. Let $C \in \mathcal{C}$. (i) S_C is nonempty, compact and convex. (ii) Since for any $i \in C$, u_i is continuous, $\tilde{u}_C$ is continuous. (iii) Let $s_{-C} \in \boldsymbol{S}_{-C}$. For any $i \in C$, $u_i(\cdot, s_{-C})$ is concave. Note that the sum of concave functions is concave. Then, $\tilde{u}_C(\cdot, s_{-C})$ is concave, and thus, it is quasi-concave. From (i)–(iii) for each $C \in \mathcal{C}$, by Theorem 1.2 in Fudenberg and Tirole (1991), there exists a Nash equilibrium s^* in $\tilde{G}$. Let $r^* \in \boldsymbol{R}$ such that for any $C \in \mathcal{C}$ and any $i \in C$, $r_{Ci}^* = \frac{\sum_{j \in C}\left(u_j(s^*) - u_j\left(\bar{s}_C, s_{-C}^*\right)\right)}{|C|} - \left(u_i(s^*) - u_i\left(\bar{s}_C, s_{-C}^*\right)\right)$. Then, by Lemma 1, (s^*, r^*) is a Nash solution with transfers in G. Therefore, there exists a Nash solution with transfers in G. □

D Proof of Theorem 3

Let $\prec$ be the binary relation on N such that for any $i, j \in N$, $i \prec j$ if and only if $i \precsim j$ and $i \neq j$. For any player i, any information set I of player i, any stationary strategy tuple σ, any strategy tuple σ' equivalent to σ except for player i's action at I (σ' may be equal to σ), player i's payoff at I by σ' under player i's belief consistent with σ is calculated on the basis of the following outcome in the minimum subgame including I: in the session including I, the actions before I are the actions reaching I, the action at I is the action specified by σ', and the actions after I are induced by σ; in the other sessions in the round including I, the outcomes are induced by σ (because player i's belief is consistent with σ); in the subsequent rounds, the outcomes are induced by σ. In the following, we use this fact.

D.1 Proof of (i)

Let (s^*, r^*) be a Nash solution with transfers in G. Construct a strategy tuple family as follows. Let $\tilde{r} \in \left(\prod_{C\in\mathcal{C}} \mathbb{R}^C\right)^{[0,1)}$ such that for any $\delta \in [0,1)$, any $C \in \mathcal{C}$ and any $j \in C$,

$$\tilde{r}^{\delta}_{Cj} = \frac{\delta}{\delta|C| + (1-\delta)} \sum_{k\in C} \left(u_k(s^*) - u_k\left(\bar{s}_C, s^*_{-C}\right)\right) - \left(u_j(s^*) - u_j\left(\bar{s}_C, s^*_{-C}\right)\right).$$

Let $\hat{r} \in \left(\prod_{C\in\mathcal{C}} (R_C)^C\right)^{[0,1)}$ such that for any $\delta \in [0,1)$, any $C \in \mathcal{C}$ and any $i \in C$, $\hat{r}^{\delta i}_C \in R_C$ such that for any any $j \in C\setminus\{i\}$, $\hat{r}^{\delta i}_{Cj} = \tilde{r}^{\delta}_{Cj}$. Let $\sigma \in \Sigma^{[0,1)}$ such that for any $\delta \in [0,1)$, σ^δ is a pair of strategy tuple and consistent belief system in Γ^δ such that for any $C \in \mathcal{C}$ and any $i \in C$, in any session with C, if player i is a proposer, she proposes $\left(s^*_C, \hat{r}^{\delta i}_C\right)$; if she is a responder, player i accepts a proposal (s_C, r_C) if and only if for any responder j such that $i \precsim j$,

$$\left(u_j\left(s_C, s^*_{-C}\right) + r_{Cj}\right) + \frac{\delta}{1-\delta}\left(u_j(s^*) + \tilde{r}^{\delta}_{Cj}\right) \geq u_j\left(\bar{s}_C, s^*_{-C}\right) + \frac{\delta}{1-\delta}\left(u_j(s^*) - \sum_{k\in C\setminus\{j\}} \tilde{r}^{\delta}_{Ck}\right), \tag{1}$$

i.e.,

$$u_j\left(s_C, s^*_{-C}\right) + r_{Cj} \geq u_j\left(\bar{s}_C, s^*_{-C}\right) + \frac{\delta}{\delta|C| + (1-\delta)} \sum_{k\in C} \left(u_k(s^*) - u_k\left(\bar{s}_C, s^*_{-C}\right)\right). \tag{2}$$

Let $\delta \in [0,1)$. Then, σ^δ is a no-delay constant-alternative SPBE as follows. By the construction of σ, σ^δ is stationary and constant-alternative. For any $C \in \mathcal{C}$ and any $i \in C$ and any $j \in C\setminus\{i\}$, since

$$\left(u_i\left(s^*_C, s^*_{-C}\right) + \hat{r}^{\delta i}_{Cj}\right) - \left(u_i\left(\bar{s}_C, s^*_{-C}\right) + \frac{\delta}{\delta|C| + (1-\delta)} \sum_{k\in C} \left(u_k(s^*) - u_k\left(\bar{s}_C, s^*_{-C}\right)\right)\right) = 0,$$

player i's proposal by σ^δ in any session with C is accepted by all responders in σ^δ. The belief system of σ^δ is constructed as consistent with the strategy tuple of σ^δ. The following Lemmas 2 and 3 show the unimprovability of responses and proposals by

σ^δ, respectively, and thus, the strategy tuple of σ^δ is sequentially rational.

Lemma 2 (Unimprovability of responses by σ^δ)**.** *Consider a information set where any player i is asked to respond to any proposal (s_C, r_C) in any session with any $C \in \mathcal{C}$. Then, player i's response by σ^δ is unimprovable.*

Proof. Consider the case where player i's response by σ^δ is acceptance. Then, for any responder j such that $i \precsim j$, (1) holds. Thus, for any responder j such that $i \precsim j$ accepts (s_C, r_C) in σ^δ. Hence, player i's payoff by σ^δ is

$$\left(u_i\left(s_C, s^*_{-C}\right) + r_{Ci}\right) + \frac{\delta}{1-\delta}\left(u_i\left(s^*\right) + \tilde{r}^\delta_{Ci}\right). \tag{3}$$

Player i's payoff by the one-stage deviation to rejecting (s_C, r_C) is

$$u_i\left(\bar{s}_C, s^*_{-C}\right) + \frac{\delta}{1-\delta}\left(u_i\left(s^*\right) - \sum_{j\in C\setminus\{i\}} \tilde{r}^\delta_{Cj}\right). \tag{4}$$

Since (1) holds for $j = i$, (3) is greater than or equal to (4).

Consider the case where player i's response by σ^δ is rejection. Then, player i's payoff by σ^δ is (4).

Consider the subcase where for any responder j such that $i \prec j$, (1) holds, which implies that for i, (1) does not hold. Then, since any responder j such that $i \prec j$ accepts (s_C, r_C) in σ^δ, player i's payoff by the one-stage deviation to accepting (s_C, r_C) is (3). Since for i, (1) does not hold, (4) is greater than (3).

Consider the subcase where for some responder j such that $i \prec j$, (1) does not hold. Then, the successor of i in $\precsim$ rejects (s_C, r_C) in σ^δ. Thus, player i's payoff by the the one-stage deviation to accepting (s_C, r_C) is

$$u_i\left(\bar{s}_C, s^*_{-C}\right) + \frac{\delta}{1-\delta}\left(u_i\left(s^*\right) + \tilde{r}^\delta_{Ci}\right). \tag{5}$$

The difference between (4) and (5) is

$$-\frac{\delta}{1-\delta}\sum_{k\in C}\tilde{r}^{\delta}_{Ck}=\frac{\delta}{\delta\left|C\right|+(1-\delta)}\left(\sum_{k\in C}u_k\left(s^*\right)-\sum_{k\in C}u_k\left(\bar{s}_C,s^*_{-C}\right)\right),$$

which is greater than or equal to 0 because s^* is a Nash solution with transfers in G. □

Lemma 3 (Unimprovability of proposals by σ^δ)**.** *Consider a information set where any player i offers a proposal in any session with any $C\in\mathcal{C}$. Then, player i's proposal by σ^δ is unimprovable.*

Proof. Player i's payoff by σ^δ is

$$\frac{1}{1-\delta}\left(u_i\left(s^*\right)-\sum_{j\in C\setminus\{i\}}\tilde{r}^{\delta}_{Cj}\right).$$

Proposer i's payoff by the one-stage deviation to proposing (s_C,r_C) that is accepted by all responders in σ^δ is

$$\left(u_i\left(s_C,s^*_{-C}\right)-\sum_{j\in C\setminus\{i\}}r_{Cj}\right)+\frac{\delta}{1-\delta}\left(u_i\left(s^*\right)-\sum_{j\in C\setminus\{i\}}\tilde{r}^{\delta}_{Cj}\right).$$

The gain g from this deviation is

$$g=u_i\left(s_C,s^*_{-C}\right)-\sum_{j\in C\setminus\{i\}}r_{Cj}-u_i\left(s^*\right)+\sum_{j\in C}\tilde{r}^{\delta}_{Cj}-\tilde{r}^{\delta}_{Ci}.$$

Note that (s_C,r_C) is accepted in σ^δ, for any $j\in C\setminus\{i\}$, (1) holds, and thus,

$$\sum_{j\in C\setminus\{i\}}r_{Cj}\geq-\sum_{j\in C\setminus\{i\}}u_j\left(s_C,s^*_{-C}\right)+\sum_{j\in C\setminus\{i\}}u_j\left(\bar{s}_C,s^*_{-C}\right)-\frac{\delta\left(|C|-1\right)}{1-\delta}\sum_{j\in C}\tilde{r}^{\delta}_{Cj}.$$

Then,

$$g\leq\sum_{j\in C}\left(u_j\left(s_C,s^*_{-C}\right)-u_j\left(\bar{s}_C,s^*_{-C}\right)\right)+\frac{\delta\left(|C|-1\right)+(1-\delta)}{1-\delta}\sum_{j\in C}\tilde{r}^{\delta}_{Cj}-\left(u_i\left(s^*\right)-u_i\left(\bar{s}_C,s^*_{-C}\right)\right)-\tilde{r}^{\delta}_{Ci}.$$

Note that by the definition of $\tilde{r}^{\delta}_{Cj}$,

$$\sum_{j\in C}\tilde{r}^{\delta}_{Cj}=-\frac{1-\delta}{\delta\left|C\right|+\left(1-\delta\right)}\sum_{j\in C}\left(u_j\left(s^*\right)-u_j\left(\bar{s}_C,s^*_{-C}\right)\right). \tag{6}$$

Then,

$$g\leq\sum_{j\in C}\left(u_j\left(s_C,s^*_{-C}\right)-u_j\left(\bar{s}_C,s^*_{-C}\right)\right)-\sum_{j\in C}\left(u_j\left(s^*\right)-u_j\left(\bar{s}_C,s^*_{-C}\right)\right).$$

Note that s^* is a Nash solution in G, and thus, $\sum_{j\in C}u_j\left(s_C,s^*_{-C}\right)\leq\sum_{j\in C}u_j\left(s^*\right)$. Then, $g\leq 0$.

Proposer i's payoff by the one-stage deviation to any proposal that is rejected by player j in σ^{δ} is

$$u_i\left(\bar{s}_C,s^*_{-C}\right)+\frac{\delta}{1-\delta}\left(u_i\left(s^*\right)+\tilde{r}^{\delta}_{Ci}\right).$$

The gain g from this deviation is

$$g=u_i\left(\bar{s}_C,s^*_{-C}\right)-u_i\left(s^*\right)-\tilde{r}^{\delta}_{Ci}+\frac{1}{1-\delta}\sum_{j\in C}\tilde{r}^{\delta}_{Ci}.$$

Thus, by (6),

$$g=-\frac{1+\delta}{\delta\left|C\right|+\left(1-\delta\right)}\sum_{j\in C}\left(u_j\left(s^*\right)-u_j\left(\bar{s}_C,s^*_{-C}\right)\right).$$

Note that s^* is a Nash solution in G, and thus, $\sum_{j\in C}u_j\left(s_C,s^*_{-C}\right)\leq\sum_{j\in C}u_j\left(s^*\right)$. $g\leq 0$. □

Since for any $C\in\mathcal{C}$ and any $i\in C$,

$$\begin{aligned}\lim_{\delta\to 1}r^{\delta}_{Ci}&=\lim_{\delta\to 1}\left(\frac{\delta}{\delta\left|C\right|+\left(1-\delta\right)}\sum_{j\in C}\left(u_j\left(s^*\right)-u_j\left(\bar{s}_C,s^*_{-C}\right)\right)-\left(u_i\left(s^*\right)-u_i\left(\bar{s}_C,s^*_{-C}\right)\right)\right)\\&=\frac{1}{\left|C\right|}\sum_{j\in C}\left(u_j\left(s^*\right)-u_j\left(\bar{s}_C,s^*_{-C}\right)\right)-\left(u_i\left(s^*\right)-u_i\left(\bar{s}_C,s^*_{-C}\right)\right).\end{aligned}$$

By Lemma 1, $\lim_{\delta \to 1} r^{\delta}_{Ci} = r^{*}_{Ci}$. Therefore, σ supports (s^*, r^*). □

D.2 Proof of (ii)

Let σ be a no-delay constant-alternative SPBE family. Let (s, r) be the pair of $s \in \left(\prod_{C \in \mathcal{C}} (S_C)^C\right)^{[0,1)}$ and $r \in \left(\prod_{C \in \mathcal{C}} (R_C)^C\right)^{[0,1)}$ such that for any $\delta \in [0, 1)$, any $C \in \mathcal{C}$ and any $i \in C$, $(s^{\delta i}_C, r^{\delta i}_C)$ is player i's proposal by σ^δ in any session with C of any round with any p such that $p_C = i$ in Γ^δ.

Let $\delta \in [0, 1)$. Let $C \in \mathcal{C}$. Let p be a state. For any $i \in C$, let $\tilde{u}_i : S_C \to \mathbb{R}$ such that for any $s_C \in S_C$, $\tilde{u}_i(s_C) = u_i\left(s_C, \left(s^{\delta p_D}_D\right)_{D \in \mathcal{C} \setminus \{C\}}\right)$. For any $i \in C$, let $\tilde{p}^i$ be the state such that $\tilde{p}^i_C = i$ and for any $D \in \mathcal{C} \setminus \{C\}$, $\tilde{p}^i_D = p_D$. Proposals by σ^δ are derived as follows: Lemmas 4 and 5 provide sufficient conditions for proposals to be accepted and rejected in σ^δ, respectively; by Lemmas 4 and 5, Lemma 6 derives alternatives proposed in σ^δ; by Lemmas 4 and 5, Lemma 7 shows that each responder's payoffs by acceptance and rejection are equal, and by Lemma 7, Lemma 8 derives alternatives proposed in σ^δ.

Lemma 4 (Sufficient condition for proposals to be accepted in σ^δ)**.** *For any $i \in C$, in any session with C of any round with $\tilde{p}^i$, any proposal (s_C, r_C) such that for any $j \in C \setminus \{i\}$,*

$$\left(\tilde{u}_j(s_C) + r_{Cj}\right) + \frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s^{\delta i}_C\right) + r^{\delta i}_{Cj}\right) > \tilde{u}_i(\bar{s}_C) + \frac{\delta}{1-\delta}\left(\tilde{u}_i\left(s^{\delta j}_C\right) + r^{\delta j}_{Cj}\right)$$

is accepted by all responders in σ^δ.

Proof. Let j be a responder. Suppose that the all responders that follow responder j in $\precsim$ accept (s_C, r_C) in σ. Then, responder j's payoff by acceptance and rejection given the other actions in σ^δ are

$$\left(\tilde{u}_j(s_C) + r_{Cj} + \sum_{j \in D \in \mathcal{C} \setminus \{C\}} r^{\delta p_D}_{Dj}\right) + \frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s^{\delta i}_C\right) + r^{\delta i}_{Cj} + \sum_{j \in D \in \mathcal{C} \setminus \{C\}} r^{\delta p_D}_{Dj}\right)$$

and

$$\left(\tilde{u}_j\left(\bar{s}_C\right)+\sum_{j\in D\in\mathcal{C}\setminus\{C\}} r_{Dj}^{\delta p_D}\right)+\frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta j}\right)+r_{Cj}^{j}+\sum_{j\in D\in\mathcal{C}\setminus\{C\}} r_{Dj}^{\delta p_D}\right),$$

respectively. By the supposition, the former is greater than the latter. Thus, she accepts the proposal by σ^δ. Then, by the mathematical induction, all responders accept the proposal. □

Lemma 5 (Sufficient condition for proposals to be rejected in σ^δ)**.** *For any $i \in C$, in any session with C of any round with p^i, any proposal (s_C, r_C) such that for some $j \in C \setminus \{i\}$,*

$$\left(\tilde{u}_j\left(s_C\right)+r_{Cj}\right)+\frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta i}\right)+r_{Cj}^{\delta i}\right)<\tilde{u}_i\left(\bar{s}_C\right)+\frac{\delta}{1-\delta}\left(\tilde{u}_i\left(s_C^{\delta j}\right)+r_{Cj}^{\delta j}\right)$$

is rejected by some responder in σ^δ.

Proof. Suppose that the proposal is accepted by all responders in σ^δ. By one-stage deviation from acceptance to rejection, responder j's payoff changes from

$$\left(\tilde{u}_j\left(s_C\right)+r_{Cj}+\sum_{j\in D\in\mathcal{C}\setminus\{C\}} r_{Dj}^{\delta p_D}\right)+\frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta i}\right)+r_{Cj}^{\delta i}+\sum_{j\in D\in\mathcal{C}\setminus\{C\}} r_{Dj}^{\delta p_D}\right)$$

to

$$\left(\tilde{u}_j\left(\bar{s}_C\right)+\sum_{j\in D\in\mathcal{C}\setminus\{C\}} r_{Dj}^{\delta p_D}\right)+\frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta j}\right)+r_{Cj}^{\delta j}+\sum_{j\in D\in\mathcal{C}\setminus\{C\}} r_{Dj}^{\delta p_D}\right).$$

By the supposition, this change is improvement, which is a contradiction. □

Lemma 6 (Alternatives proposed in σ^δ)**.** *For any $i \in C$, $s_C^{\delta i} \in \max_{s_C\in S_C} \sum_{j\in C} \tilde{u}_j\left(s_C\right)$.*

Proof. Suppose that $s_C^{\delta i} \notin \max_{s_C\in S_C} \sum_{j\in C} \tilde{u}_j\left(s_C\right)$. Then, for some $s_C \in S_C$, $\sum_{j\in C} \tilde{u}_j\left(s_C\right) > \sum_{j\in C} \tilde{u}_j\left(s_C^{\delta i}\right)$. Let $r_C \in R_C$ such that for any $j \in C \setminus \{i\}$,

$$r_{Cj}=r_{Cj}^{\delta i}-\left(\tilde{u}_j\left(s_C\right)-\tilde{u}_j\left(s_C^{\delta i}\right)\right)+\frac{\sum_{k\in C}\tilde{u}_k\left(s_C\right)-\sum_{k\in C}\tilde{u}_k\left(s_C^{\delta i}\right)}{|C|}.$$

Since σ^δ involves no delay, $\left(s_C^{\delta i}, r_C^{\delta i}\right)$ is accepted by all responders in σ^δ. Thus, by Lemma 5, for any $j \in C \setminus \{i\}$,

$$\left(\tilde{u}_j\left(s_C^{\delta i}\right) + r_{Cj}^{\delta i}\right) + \frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta i}\right) + r_{Cj}^{\delta i}\right) \geq \tilde{u}_j\left(\bar{s}_C\right) + \frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta j}\right) + r_{Cj}^{\delta j}\right)$$

Note that by the definition of r_C, for any $j \in C \setminus \{i\}$, $\tilde{u}_j\left(s_C\right) + r_{Cj} > \tilde{u}_j\left(s_C^{\delta i}\right) + r_{Cj}^{\delta i}$. Then, for any $j \in C \setminus \{i\}$,

$$\left(\tilde{u}_j\left(s_C\right) + r_{Cj}\right) + \frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta i}\right) + r_{Cj}^{\delta i}\right) > \tilde{u}_j\left(\bar{s}_C\right) + \frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta j}\right) + r_{Cj}^{\delta j}\right).$$

Thus, by Lemma 4, in any session with C of any round with $\tilde{p}^i$, player i's proposal $\left(s_C, r_C\right)$ is accepted by all responders in σ^δ. Hence, in a subgame starting with state $\tilde{p}^i$, by the deviation to proposing $\left(s_C, r_C\right)$ in every session with C, player i's payoff changes from $\frac{1}{1-\delta}\left(\tilde{u}_i\left(s_C^{\delta i}\right) + r_{Ci}^{\delta i} + \sum_{i \in D \in \mathcal{C} \setminus \{C\}} r_{Di}^{\delta p_D}\right)$ to $\frac{1}{1-\delta}\left(\tilde{u}_i\left(s_C\right) + r_{Ci} + \sum_{i \in D \in \mathcal{C} \setminus \{C\}} r_{Di}^{\delta p_D}\right)$. Since by the definition of r_C,

$$\begin{aligned}
&\left(\tilde{u}_i\left(s_C\right) + r_{Ci}\right) - \left(\tilde{u}_i\left(s_C^{\delta i}\right) + r_{Ci}^{\delta i}\right) \\
&= \left(\tilde{u}_i\left(s_C\right) - \tilde{u}_i\left(s_C^{\delta i}\right)\right) - \left(\sum_{k \in C \setminus \{i\}} r_{Ck} - \sum_{k \in C \setminus \{i\}} r_{Ck}^{\delta i}\right) \\
&= \left(\tilde{u}_i\left(s_C\right) - \tilde{u}_i\left(s_C^{\delta i}\right)\right) + \sum_{k \in C \setminus \{i\}}\left(\tilde{u}_k\left(s_C\right) - \tilde{u}_k\left(s_C^{\delta i}\right)\right) - \left(|C| - 1\right)\frac{\sum_{k \in C} \tilde{u}_k\left(s_C\right) - \sum_{k \in C} \tilde{u}_k\left(s_C^{\delta i}\right)}{|C|} \\
&= \frac{\sum_{k \in C} \tilde{u}_k\left(s_C\right) - \sum_{k \in C} \tilde{u}_k\left(s_C^{\delta i}\right)}{|C|} > 0,
\end{aligned}$$

this change is improvement, which is a contradiction. □

Lemma 7 (Bindingness of condition for proposals to be accepted in σ^δ)**.** *For any distinct $i, j \in C$,*

$$\frac{1}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta i}\right) + r_{Cj}^{\delta i}\right) = \tilde{u}_j\left(\bar{s}_C\right) + \frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta j}\right) + r_{Cj}^{\delta j}\right). \tag{7}$$

Proof. Since in any session with C of any round with $\tilde{p}^i$, player i's proposal $\left(s_C^{\delta i}, r_C^{\delta i}\right)$ is accepted by all responders in σ^δ, by Lemma 5, for any $j \in C \setminus \{i\}$,

$$\frac{1}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta i}\right)+r_{Cj}^{\delta i}\right) \geq \tilde{u}_j\left(\bar{s}_C\right)+\frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta j}\right)+r_{Cj}^{\delta j}\right). \tag{8}$$

Suppose that for some $k \in C \setminus \{i\}$, this inequality strictly holds. Let

$$\alpha := \frac{\frac{1}{1-\delta}\left(\tilde{u}_k\left(s_C^{\delta i}\right)+r_{Ck}^{\delta i}\right)-\left(\tilde{u}_k\left(\bar{s}_C\right)+\frac{\delta}{1-\delta}\left(\tilde{u}_k\left(s_C^{\delta k}\right)+r_{Ck}^{\delta k}\right)\right)}{|C|}.$$

Let r_C be the element in R_C such that $r_{Ck} = r_{Ck}^{\delta i} - (|C|-1)\alpha$ and for any $j \in C \setminus \{k, i\}$, $r_{Cj} = r_{Cj}^{\delta i} + \alpha$. Since $\alpha > 0$,

$$\left(\left(\tilde{u}_k\left(s_C^{\delta i}\right)+r_{Ck}\right)+\frac{\delta}{1-\delta}\left(\tilde{u}_k\left(s_C^{\delta i}\right)+r_{Ck}^{\delta i}\right)\right)-\left(\tilde{u}_k\left(\bar{s}_C\right)+\frac{\delta}{1-\delta}\left(\tilde{u}_k\left(s_C^{\delta k}\right)+r_{Ck}^{\delta k}\right)\right)$$
$$=\alpha > 0.$$

For any $j \in C \setminus \{k, i\}$, since $r_{Cj} = r_{Cj}^{\delta i} + \alpha > r_{Cj}^{\delta i}$ and (8) holds,

$$\left(\tilde{u}_j\left(s_C^{\delta i}\right)+r_{Cj}\right)+\frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta i}\right)+r_{Cj}^{\delta i}\right) > \tilde{u}_j\left(\bar{s}_C\right)+\frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta j}\right)+r_{Cj}^{\delta j}\right).$$

Thus, by Lemma 4, in any session with C of any round with $\tilde{p}^i$, player i's proposal $\left(s_C^{\delta i}, r_C\right)$ is accepted by all responders in σ^δ. Thus, in a subgame starting with state $\tilde{p}^i$, by the the deviation to proposing $\left(s_C^{\delta i}, r_C\right)$ in every session with C, player i's payoff changes from $\frac{1}{1-\delta}\left(\tilde{u}_i\left(s_C^{\delta i}\right)+r_{Ci}^{\delta i}+\sum_{i\in D\in\mathcal{C}\setminus\{C\}} r_{Di}^{\delta p_D}\right)$ to $\frac{1}{1-\delta}\left(\tilde{u}_i\left(s_C^{\delta i}\right)+r_{Ci}+\sum_{i\in D\in\mathcal{C}\setminus\{C\}} r_{Di}^{\delta p_D}\right)$. Note that $r_{Ci} = -\sum_{j\in C\setminus\{i,k\}} r_{Cj} - r_{Ck} = -\sum_{j\in C\setminus\{i,k\}}\left(r_{Cj}^{\delta i}+\alpha\right) - \left(r_{Ck}^{\delta i} - (|C|-1)\alpha\right) = -\sum_{j\in C\setminus\{i\}} r_{Cj}^{\delta i} + \alpha = r_{Ci}^{\delta i} + \alpha > r_{Ci}^{\delta i}$. Then, this change is improvement, which is a contradiction. □

Lemma 8 (Transfers proposed in σ^δ)**.** *For any distinct $i, j \in C$,*

$$r_{Cj}^{\delta i} = \frac{\delta}{\delta|C|+(1-\delta)}\sum_{k\in C}\left(\tilde{u}_k\left(s_C^{\delta i}\right)-\tilde{u}_k\left(\bar{s}_C\right)\right)-\left(\tilde{u}_j\left(s_C^{\delta i}\right)-\tilde{u}_j\left(\bar{s}_C\right)\right).$$

Proof. By Lemma 7, for any $j \in C$ and any $h, i \in C \setminus \{j\}$, $\tilde{u}_j\left(s_C^{\delta h}\right) + r_{Cj}^{\delta h} = \tilde{u}_j\left(s_C^{\delta i}\right) + r_{Cj}^{\delta i}$.

Hence, there exists $v_C \in \mathbb{R}^C$ such that for any distinct $i, j \in C$, $v_{Cj} = \tilde{u}_j\left(s_C^{\delta i}\right) + r_{Cj}^{\delta i}$. Therefore, by Lemma 7, for any $j \in C$,

$$\begin{aligned}
\frac{1}{1-\delta} v_{Cj} &= \tilde{u}_j\left(\bar{s}_C\right) + \frac{\delta}{1-\delta}\left(\tilde{u}_j\left(s_C^{\delta j}\right) + r_{Cj}^{\delta j}\right) \\
&= \tilde{u}_j\left(\bar{s}_C\right) + \frac{\delta}{1-\delta}\left(\sum_{k \in C} \tilde{u}_k\left(s_C^{\delta j}\right) - \sum_{k \in C \setminus \{j\}} \tilde{u}_k\left(s_C^{\delta j}\right) - \sum_{k \in C \setminus \{j\}} r_{Ck}^{\delta j}\right) \\
&= \tilde{u}_j\left(\bar{s}_C\right) + \frac{\delta}{1-\delta}\left(\sum_{k \in C} \tilde{u}_k\left(s_C^{\delta j}\right) - \sum_{k \in C \setminus \{j\}} v_{Ck}\right),
\end{aligned}$$

and thus,

$$v_{Cj} = \tilde{u}_j\left(\bar{s}_C\right) + \frac{\delta}{1-\delta}\left(\sum_{k \in C} \tilde{u}_k\left(s_C^{\delta j}\right) - \sum_{k \in C} v_{Ck}\right). \tag{9}$$

Hence, by summing this equation with respect to j over C and Lemma 6, for any $j \in C$,

$$\sum_{k \in C} v_{Ck} = \sum_{k \in C} \tilde{u}_k\left(\bar{s}_C\right) + \frac{\delta |C|}{1-\delta}\left(\sum_{k \in C} \tilde{u}_k\left(s_C^{\delta j}\right) - \sum_{k \in C} v_{Ck}\right),$$

and thus,

$$\sum_{k \in C} v_{Ck} = \frac{1}{\delta |C| + (1-\delta)}\left(\delta |C| \sum_{k \in C} \tilde{u}_k\left(s_C^{\delta j}\right) + (1-\delta) \sum_{k \in C} \tilde{u}_k\left(\bar{s}_C\right)\right),$$

By substituting this into (9) and Lemma 6, for any distinct $i, j \in C$,

$$v_{Cj} = \frac{\delta}{\delta |C| + (1-\delta)}\left(\sum_{k \in C} \tilde{u}_k\left(s_C^{\delta j}\right) - \sum_{k \in C} \tilde{u}_k\left(\bar{s}_C\right)\right) + \tilde{u}_j\left(\bar{s}_C\right),$$

and thus,

$$r_{Cj}^i = \frac{\delta}{\delta |C| + (1-\delta)} \sum_{k \in C}\left(\tilde{u}_k\left(s_C^{\delta i}\right) - \tilde{u}_k\left(\bar{s}_C\right)\right) - \left(\tilde{u}_j\left(s_C^{\delta i}\right) - \tilde{u}_j\left(\bar{s}_C\right)\right).$$

□

Let $(\hat{s}, \hat{r})$ be the outcome of σ. Since σ is a no-delay constant-alternative SPBE family, there exists $s^* \in S$ such that for any $\delta \in [0,1)$ and any $t \in \mathbb{N}$, $\hat{s}^{\delta t} = s^*$. Let $r^* \in R$ such that for any $C \in \mathcal{C}$ and any $i \in C$, $r^*_{Ci} = \frac{\sum_{j\in C}\left(u_j(s^*)-u_j\left(\bar{s}_C,s^*_{-C}\right)\right)}{|C|} - \left(u_i\left(s^*\right) - u_i\left(\bar{s}_C, s^*_{-C}\right)\right)$. By Lemma 6, for any $\delta \in [0,1)$, any $t \in \mathbb{N}$ and any $C \in \mathcal{C}$, $\hat{s}^{\delta t}_C \in \arg\max_{s_C \in S_C} \sum_{i\in C} u_i\left(s_C, s^{\delta t}_{-C}\right)$. Thus, for any $C \in \mathcal{C}$, $s^*_C \in \arg\max_{s_C\in S_C} \sum_{i\in C} u_i\left(s_C, s^*_{-C}\right)$. Hence, by Lemma 1, (s^*, r^*) is a Nash solution with transfers in G. By Lemma 8, for any $\delta \in [0,1)$, any $t \in \mathbb{N}$, any $C \in \mathcal{C}$ and any $i \in C$,

$$\hat{r}^{\delta t}_{Ci} = \frac{\delta}{\delta\,|C| + (1-\delta)} \sum_{j\in C} \left(u_j\left(\hat{s}^{\delta t}\right) - u_j\left(\bar{s}_C, \hat{s}^{\delta t}_{-C}\right)\right) - \left(u_i\left(\hat{s}^{\delta t}\right) - u_i\left(\bar{s}_C, \hat{s}^{\delta t}_{-C}\right)\right),$$

and thus,

$$\hat{r}^{\delta t}_{Ci} = \frac{\delta}{\delta\,|C| + (1-\delta)} \sum_{j\in C} \left(u_j\left(s^*\right) - u_j\left(\bar{s}_C, s^*_{-C}\right)\right) - \left(u_i\left(s^*\right) - u_i\left(\bar{s}_C, s^*_{-C}\right)\right).$$

Hence, for any $C \in \mathcal{C}$, any $t \in \mathbb{N}$ and any $i \in C$, $\lim_{\delta\to 1} \hat{r}^{\delta t}_{Ci} = r^*_{Ci}$. For any $t \in \mathbb{N}$ and any $\delta \in [0,1)$, $s^{\delta t} = s^*$. Therefore, σ supports (s^*, r^*). □

References

E. Bennett. Multilateral bargaining problems. *Games and Economic Behavior*, 19(2):151–179, 1997.

K. Binmore, A. Rubinstein, and A. Wolinsky. The Nash bargaining solution in economic modelling. *Rand Journal of Economics*, 17(2):176–188, 1986.

S. Chakrabarti, R. P. Gilles, and E. A. Lazarova. Strategic behavior under partial cooperation. *Theory and Decision*, 71(2):175–193, 2011.

T. Chipty and C. M. Snyder. The role of firm size in bilateral bargaining: A study of the cable television industry. *Review of Economics and Statistics*, 81(2):326–340, 1999.

P. Dasgupta and E. Maskin. The existence of equilibrium in discontinuous economic games, I: Theory. *Review of Economic Studies*, 53(1):1–26, 1986.

C. Davidson. Multiunit bargaining in oligopolistic industries. *Journal of Labor Economics*, 6(3):397–422, 1988.

D. Fudenberg and J. Tirole. *Game Theory*. MIT Press, 1991.

G. Genicot and D. Ray. Contracts and externalities: How things fall apart. *Journal of Economic Theory*, 131(1):71–100, 2006.

H. Horn and A. Wolinsky. Bilateral monopolies and incentives for merger. *RAND Journal of Economics*, 19(3):408–419, 1988.

M. O. Jackson and A. Wolinsky. A strategic model of social and economic networks. *Journal of Economic Theory*, 71(1):44–74, 1996.

M. Manea. Bargaining in stationary networks. *American Economic Review*, 101(5):2042–2080, 2011.

N. Matsushima and R. Shinohara. Pre-negotiation commitment and internalization in public good provision through bilateral negotiations. Discussion Paper No. 948, Institute of Social and Economic Research, Osaka University, 2015.

J. Nash. Non-cooperative games. *Annals of Mathematics*, 54(2):286–295, 1951.

J. F. Nash, Jr. The bargaining problem. *Econometrica*, 18(2):155–162, 1950a.

J. F. Nash, Jr. Equilibrium points in *n*-person games. *Proceedings of the National Academy of Sciences of the United States of America*, 36(1):48–49, 1950b.

J. F. Nash, Jr. Two-person cooperative games. *Econometrica*, 21(1):128–140, 1953.

A. Okada. A noncooperative coalitional bargaining game with random proposers. *Games and Economic Behavior*, 16(1):97–108, 1996.

A. Okada. The Nash bargaining solution in general *n*-person cooperative games. *Journal of Economic Theory*, 145(6):2356–2379, 2010.

A. Rubinstein. Perfect equilibrium in a bargaining model. *Econometrica*, 50(1):97–110, 1982.

D. M. Topkis. Equilibrium points in nonzero-sum *n*-person submodular games. *SIAM Journal on Control and Optimization*, 17(6):773–787, 1979.

L. Zhao. Cross-hauling direct foreign investment and unionized oligopoly. *European Economic Review*, 39(6):1237–1253, 1995.