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DISCUSSION PAPER

Bargaining Delay under Partial Breakdowns, Externalities and Nontransferable Utilities

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Abstract

Based on a nontransferable-utility partition function form game, we define a propose-respond bargaining game in extensive form, where the rejecter of a proposal exits from the game with a positive probability. We present an example in which delay of agreements occurs. We also provide sufficient conditions for no delay.

Keywords: Partition function form game; Nontransferable utility; Rejecter-exit partial breakdown; Delay

JEL classification codes: C72; C73; C78

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1 Introduction

Delay of agreements is an important issue in bargaining theory. This is because delays are a source of inefficiency due to time discounting or other factors. Rubinstein (1982) showed that no delay occurs in a unique subgame perfect equilibrium in bilateral bargaining. Since then, the robustness of this no-delay result has been examined extensively in the literature.

In complete-information multilateral bargaining, delay may occur. Several studies have shown that delay may occur in a nonstationary subgame perfect equilibrium (e.g., see Osborne and Rubinstein (1990)). Chatterjee et al. (1993) showed that delay may occur in a stationary subgame perfect equilibrium (SSPE), in coalitional bargaining[1] with the rejecter-propose protocol;[2] Britz et al. (2015), under a general bargaining protocol that incorporates various protocols from previous studies such as Hart and Mas-Colell (1996), Kultti and Vartiainen (2010) and Laruelle and Valenciano (2008); Miyakawa (2009), under the nonsuperadditive environment; Merlo and Wilson (1995), in unanimity bargaining[3] in which the size of the cake changes stochastically over time.

However, under the random-proposer protocol,[4] the superadditive environment and the deterministic cake sizes, no delay occurs in any SSPE. Okada (1996) showed that delay does not occur in any SSPE, under no externality and transferable utilities; Hart and Mas-Colell (1996), under no externality and nontransferable utilities; Gomes (2005), under externalities and transferable utilities; Okada (2010), when the underlying game is a normal form game. Britz et al. (2010) pointed out that the time-invariant property of the random-proposer protocol is important for the no-delay result.

In contrast, we show that even under the random-proposer protocol, the superadditive environment and the deterministic cake sizes, if a *rejecter-exit partial breakdown* occurs with even a small probability, delay may occur in an SSPE in

[1]In coalitional bargaining, a proposer chooses a subset of the player set (a coalition) to make an offer, and the approval by all members in the chosen coalition is required for the proposal to be agreed.

[2]This protocol is such that the first rejecter of a proposal becomes the proposer in the next round.

[3]In unanimity bargaining, there is no choice of a coalition, and the approval by all players is required.

[4]The random-proposer protocol is such that the proposer is selected according to some probability independent of history.

unanimity bargaining. The rejecter-exit partial breakdown describes a situation where the rejecter of a proposal exits from bargaining. One rationale for this partial breakdown is that the rejection of a proposal causes the proposer's hostility toward the rejecter, and the rejecter is forced to exit from bargaining. We display that a combination of the partial breakdown and externalities causes delay of agreements, which is an unknown aspect of the bargaining delay. Under the rejecter-exit partial breakdown, some proposer may offer an unacceptable proposal and make a player exit in order to (i) remove a strong player and/or (ii) remove a player, commit himself/herself not to form a coalition with this player and prevent a third player from enjoying a positive externality from this coalition. We show such a delay under transferable utilities, which implies that nontransferable utilities are not necessary for delay.

Moreover, we provide a sufficient condition for no delay to occur in any SSPE under externalities and nontransferable utilities. We also show that no delay occurs in any SSPE under no externality, which implies that externalities are necessary for delay.

Delay may occur even if players bargain over coalitions as well as allocations. In the current study, players do not bargain over coalitions, and only the coalition consisting of all active players is formed when an agreement is achieved. Thus, players have no choice but to remove a player by an unacceptable proposal in order to form a subcoalition. Hence, one may think that if the model is modified by allowing players to bargain over coalitions, the players propose subcoalitions rather than propose an unacceptable proposal and remove a player, and delay does not occur. However, this is not true. In the working paper version, Kawamori and Miyakawa (2012), players are allowed to bargain over coalitions, but delay occurs under some partition function with positive externalities. The reason is as follows: when a player proposes a target subcoalition, a coalition partner can, by rejecting the proposal and exiting, enjoy a positive externality from the coalition formed by the other remaining players, and thus, he/she in a strong bargaining position; meanwhile, when the player removes a player by an unacceptable proposal and commits to not forming a coalition with this player and he/she proposes the target subcoalition in the next round, the coalition partner cannot enjoy the positive externality.

Kawamori and Miyakawa (2016) considered coalitional bargaining with rejecter-exit partial breakdowns but did not examine delay. The current study presents an example of delay, whereas Kawamori and Miyakawa (2016) did not present an example of delay. The current study presents a condition for *any* SSPE to have the no-delay property, whereas Kawamori and Miyakawa (2016) presented a condition for *some* SSPE to have the full-coalition and no-delay property (the property such that the coalition of all active players is immediately formed in any subgame).

In the current study and Jehiel and Moldovanu (1995), opposite externalities cause delay. Jehiel and Moldovanu (1995) showed that delay may occur owing to negative externalities and does not occur under positive externalities in a random-matching setting. In their example of delays, a war of attrition takes place between a strong proposer and weak responders being afraid of negative externalities. Their logic is completely different from ours because positive externalities play an important role in our example of delay.

In the example of our paper and Example 1 (4-player example) of Chatterjee et al. (1993), the proposal of a weak player (say player 1) is rejected by a player (say player 2) who not only is strong but also makes another player (say player 3) strong. In our paper, player 2 makes player 3 strong through a positive externality from $\{1, 2\}$; in Chatterjee et al. (1993), player 2 makes player 3 strong through an opportunity of formation of $\{2, 3\}$ having a large worth. In our paper, player 1 makes player 2 reject a proposal and exit from the negotiation and prevents player 3 from enjoying the positive externality from $\{1, 2\}$; in Chatterjee et al. (1993), player 1 makes player 2 reject a proposal, form $\{2, 4\}$ in the next round and exit from the negotiation and prevents player 3 from enjoying an opportunity from formation of $\{2, 3\}$ and $\{3, 4\}$ having large worths.

The rejecter-exit partial breakdown is the most important feature of our model. A partial breakdown describes a situation where some players exit randomly from bargaining after rejections, and the remaining players continue negotiations. A partial breakdown is different from discounting, which is considered as a situation where all players exist with some probability from bargaining. In real-life negotiations, it sometimes happens that some players are exogenously removed from the negotiating table to proceed with the negotiations. In Section 6 of Hart and Mas-Colell (1996), a broad class of partial breakdowns were considered. However, the rejecter-exist partial breakdown is definitely different from them. In our paper, probabilities of exit of players depend on the identity of the rejecter; in Hart and Mas-Colell (1996), the probabilities do not depend on the identity of the rejecter.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 provides an example of delay. Section 4 provides a sufficient condition for no delay. Section 5 discusses problems that remain. The proofs of all propositions are given in Appendix.

2 Model

For any sets X and Y, let Y^X be the set of functions from X to Y. For any function f and any x in the domain of f, let f_x be the image of x under f, i.e., $f_x := f(x)$. For any set A, a *partition* of A is π such that $\pi \not\ni \emptyset$, $S \cap T = \emptyset$ for any distinct $S, T \in \pi$ and $\bigcup \pi = A$.[5,6] For any set A, let Π^A be the set of partitions of A. For any set A, let $\mathcal{C}^A := \left\{(S, \pi) \in 2^A \times \Pi^A \mid S \in \pi\right\}$.

2.1 Partition function form game

Let (N, V) be a partition function form game: N is a nonempty finite set, and V is a function from $\mathcal{C}^N$ to $\bigcup_{S \in 2^N \setminus \{\emptyset\}} 2^{\mathbb{R}^S}$ such that for any $(S, \pi) \in \mathcal{C}^N$, $V(S, \pi) \subset \mathbb{R}^S$.

Throughout this paper, it is assumed that Assumption 1 holds. This assumption is standard in the literature.

Assumption 1. Let $(S, \pi) \in \mathcal{C}^N$.

(i) $V(S, \pi)$ is closed, convex and comprehensive (i.e., $V(S, \pi) - \mathbb{R}^S_+ \subset V(S, \pi)$)).

(ii) $0 \in V(S, \pi)$, and $V(S, \pi) \cap \mathbb{R}^S_+$ is bounded.

[5] In this study, for any set $\mathcal{A}$, $\bigcup \mathcal{A} := \{a \mid \exists A \in \mathcal{A}\, (a \in A)\}$. Some authors denote this by $\bigcup_{A \in \mathcal{A}} A$.
[6] According to this definition, the empty set is a unique partition of the empty set.

(iii) There exists a differentiable function $f : \mathbb{R}^S \to \mathbb{R}$ such that $f^{-1}(\mathbb{R}_-) = V(S,\pi)$ and $f^{-1}(\{0\}) = \mathrm{bd}(V(S,\pi))$, where $\mathrm{bd}(V(S,\pi))$ is a boundary of $V(S,\pi)$.

(iv) For any $x \in f^{-1}(\{0\})$, $\nabla f(x) \gg 0$.

(v) (N,V) is strictly superadditive, i.e., for any $\pi \in \Pi^N$ and any disjoint $S,T \in \pi$, $V(S,\pi) \times V(T,\pi) \subset \mathrm{int}(V(S \cup T, \pi \setminus \{S,T\} \cup \{S \cup T\}))$.

Assumption 2 states that (i) players' utilities are transferable and (ii) there is no externality in coalition formation.

Assumption 2. (i) There exists $v : \mathcal{C}^N \to \mathbb{R}$ such that for any $(S,\pi) \in \mathcal{C}^N$, $V(S,\pi) = \left\{x \in \mathbb{R}^S \mid \sum_{i \in S} x_i \le v(S,\pi)\right\}$.

(ii) There exists $W : 2^N \to \bigcup_{S \in 2^N \setminus \{\emptyset\}} 2^{\mathbb{R}^S}$ such that for any $(S,\pi) \in \mathcal{C}^N$, $V(S,\pi) = W(S)$.

For any $A \in 2^N \setminus \{\emptyset\}$, let $V^A : \mathcal{C}^A \to \bigcup_{S \in 2^A \setminus \{\emptyset\}} 2^{\mathbb{R}^S}$ such that for any $(S,\pi) \in \mathcal{C}^A$, $V^A(S,\pi) = V(S, \pi \cup \{\{i\} \mid i \in N \setminus A\})$, $V_+^A : \mathcal{C}^A \to \bigcup_{S \in 2^A \setminus \{\emptyset\}} 2^{\mathbb{R}^S}$ such that for any $(S,\pi) \in \mathcal{C}^A$, $V_+^A(S,\pi) = V^A(S,\pi) \cap \mathbb{R}_+^S$, and f^A be the function for $(A, \{A\} \cup \{\{i\} \mid i \in N \setminus A\})$ in Assumption 1(iii). Under Assumption 2(i), for any $A \in 2^N \setminus \{\emptyset\}$, let $v^A : \mathcal{C}^A \to \mathbb{R}$ such that for any $(S,\pi) \in \mathcal{C}^A$, $v^A(S,\pi) = v(S, \pi \cup \{\{i\} \mid i \in N \setminus A\})$. For any $A \in 2^N \setminus \{\emptyset\}$ and any $i \in A$, let $\pi_i^A := \{\{i\}, A \setminus \{i\}\} \setminus \{\emptyset\}$. For any $A \in 2^N \setminus \{\emptyset\}$, let $d^A \in \mathbb{R}^A$ such that for any $i \in A$, $d_i^A = \max V^A\left(\{i\}, \pi_i^A\right)$.

We define a Nash bargaining solution (NBS) b^A with weight tuple r^A for coalition A. For any $A \in 2^N \setminus \{\emptyset\}$, let $r^A \in \mathbb{R}_{++}^A$ such that $\sum_{i \in A} r_i^A = 1$. For any $A \in 2^N \setminus \{\emptyset\}$, if $d^A \in \mathrm{int}\left(V^A(A, \{A\})\right)$, there exists a unique solution b^A of

$$\max_{x^A \in \mathbb{R}^A} \prod_{i \in A} \left(x_i^A - d_i^A\right)^{r_i^A}$$
$$\text{s.t. } f^A(x) \le 0 \wedge \forall i \in N \left(x_i^A \ge d_i^A\right);$$

by the Krush–Kuhn–Tucker theorem, b^A is characterized as for any $i, j \in A$,

$$\frac{1}{r_i^A}\left(b_i^A - d_i^A\right)\frac{\partial f^A}{\partial x_i}\left(b^A\right) = \frac{1}{r_j^A}\left(b_j^A - d_j^A\right)\frac{\partial f^A}{\partial x_j}\left(b^A\right), \tag{1}$$

$$f^A\left(b^A\right) = 0; \tag{2}$$

under Assumption 2(i), for any $i \in A$,

$$b_i^A = r_i^A\left(v^A\left(A, \{A\}\right) - \sum_{j\in A} d_j^A\right) + d_i^A. \tag{3}$$

2.2 Extensive form game

The outline of the noncooperative bargaining game considered here is as follows. A randomly selected player proposes a payoff allocation, and the other players respond by accepting or rejecting the proposal. If all responders accept it, the proposed allocation is implemented, and the game ends. Otherwise, the first rejecter becomes inactive with certain probability or all players remain active with the remaining probability, and the active players repeat the bargaining.

For any $p \in [0, 1)$, define an extensive form game $G(p)$ as follows. A *state* is a nonempty subset of N. A state represents the set of active players. In any round with any state A, bargaining proceeds as follows.

1. Player $i \in A$ is selected with probability r_i^A.
2. Player i (say, the proposer) offers a proposal $x \in V_+^A\left(A, \{A\}\right)$.
3. Each player $j \in A \setminus \{i\}$ (say, the responder) sequentially accepts or rejects the proposal according to a predetermined order until a responder rejects it or all responders accept it.

Then, the following events occur.

- If all responders accept it, the game ends.
- Otherwise, the game proceeds to a new round
  - with state A with probability p (all active players continue the negotiation).

– with state $A \setminus \{j\}$, where j is the rejecter of the proposal, with probability $1-p$ (the rejecter becomes inactive).

If the game ends with an agreement x at state A, the payoff of any player $i \in A$ is x_i, and that of any player $j \in N \setminus A$ is $\max V(\{j\}, \{A\} \cup \{\{k\} \mid k \in N \setminus A\})$. If no agreement is achieved, any player's payoff is 0.

Definition 1 defines the stationarity of a strategy tuple.

Definition 1. A strategy tuple s is *stationary* if in s, each player's proposals depend only on states, and each player's responses depend only on states, proposers and proposals.

Definition 2 defines a strategy tuple that does not delay.

Definition 2. A strategy tuple s is a *no-delay* strategy tuple if in s, in every round, every proposal specified by s is accepted.

Remark 1. If for any state A and any partition π of A, $\prod_{B\in\pi} V^A(B,\pi) \subset \operatorname{int}\left(V^A(A,\{A\})\right)$, no-delay strategy tuples coincide with subgame-efficient strategy tuples, i.e., strategy tuples that are Pareto efficient in any subgame. Subgame-efficiency is defined by Okada (1996).

3 Delay

Example 1 is such that some player enjoys a positive externality from the coalition of the other players: $v\left(\{i\}, \pi_i^N\right) > v(\{i\}, \{\{j\} \mid j \in N\})$ $(i = 2, 3)$.

Example 1. $N = \{1, 2, 3\}$. Assumption 2(i) holds. For any $(S, \pi) \in \mathcal{C}^N$,

$$v(S,\pi) = \begin{cases} \frac{2}{5} & \text{if } (S,\pi) = \left(\{2\}, \pi_2^N\right), \left(\{3\}, \pi_3^N\right) \\ \frac{|S|-1}{2} & \text{otherwise.} \end{cases}$$

For any state A and any $i, j \in A$, $r_i^A = r_j^A$.

Proposition 1 states that in Example 1, for *any* continuation probability, there exists an SSPE that involves delay in the first round, which causes inefficiency because for any partition π of N, $\sum_{A\in\pi} v(A,\pi) < v(N\{N\})$.

Proposition 1. *Consider Example 1. For any $p \in [0,1)$, there exists an SSPE of $G(p)$ in which some player's proposal specified by this SSPE is rejected in the first round.*

Remark 2. In the proof, an SSPE where delay occurs is constructed. In this SSPE, player 1's proposal under state N is rejected by player 2, and the other proposals are accepted. Thus, the game continues to the tth period with state N with probability $\left(\frac{p}{3}\right)^{t-1}$. Thus, N is formed at the tth period ($t \geq 1$) with probability $\frac{2}{3}\left(\frac{p}{3}\right)^{t-1}$; $\{1,3\}$ is formed at $t+1$th period ($t \geq 1$) with probability $\frac{1-p}{3}\left(\frac{p}{3}\right)^{t-1}$; the other complete histories are reached with probability 0.

The intuition is as follows. By offering a proposal rejected by player 2, player 1 can make player 2 exit with probability p. The exit of player 2 has two benefits for player 1: firstly, it becomes unnecessary to allocate a payoff to player 2 with disagreement payoff $v\left(\{2\}, \pi_2^N\right)$; secondly, because player 1 commits himself/herself not to form a coalition with player 2, player 3 cannot enjoy a positive externality, and his/her disagreement payoff decreases from $v\left(\{3\}, \pi_3^N\right)$ to $v\left(\{3\}, \{\{i\} \mid i \in N\}\right)$. The exit of player 2 also has a cost for player 1: the worth available for player 1 decreases from $v(N, \{N\}) = 1$ to $v\left(\{1,3\}, \pi_2^N\right)$. The disagreement payoff of player 2 and the positive externality for player 3 are so large that the benefits dominate the cost.

4 No delay

Condition 1 means that any player's payoff in the NBS with another player removed is less than his/her payoff in some feasible and strictly individually rational allocation. Thus, under Condition 1, any player can offer an allocation such that he/she obtains more than his/her NBS payoff with any other player removed; hence, for any player, removing any other player is not profitable.

Condition 1. For any nonsingleton state A and any distinct $i, j \in A$, $\left(b_i^{A\setminus\{j\}}, d_{-i}^A\right) \in \operatorname{int}\left(V^A(A, \{A\})\right)$, and $d^A \in \operatorname{int}\left(V^A(A, \{A\})\right)$.

Remark 3. (i) Under Assumption 2(i), $\left(b_i^{A\setminus\{j\}}, d_{-i}^A\right) \in \operatorname{int}\left(V^A(A, \{A\})\right)$ is equivalent to $b_i^{A\setminus\{j\}} + \sum_{k\in A\setminus\{i\}} d_k^A < v^A(A, \{A\})$, and $d^A \in \operatorname{int}\left(V^A(A, \{A\})\right)$ is equivalent to $\sum_{k \in A} d_k^A < v^A(A, \{A\})$.

(ii) If positive externalities are more significant, d^A is larger, and thus, Condition 1 is less likely to hold. Conversely, Condition 1 is easily satisfied under negative externalities.

(iii) Under Assumption 2(i), $\left(b_i^{A\setminus\{j\}}, d_{-i}^A\right) \in \operatorname{int}\left(V^A(A, \{A\})\right)$ with int removed is equivalent to (2) in Theorem 2 in Kawamori and Miyakawa (2016), which constitutes the equivalent condition for existence of an SSPE such that the coalition of all active players is immediately formed in any subgame.

Proposition 2 states that no SSPE involves delay, (i) for a *sufficiently large* continuation probability *with* Condition 1, (ii) for *any* continuation probability *with* Condition 1 under *transferable utilities* or (iii) for *any* continuation probability *with or without* Condition 1 under *no externality*.

Proposition 2. *(i) Suppose that Condition 1 holds. Then, for some $\bar{p} \in [0, 1)$, for any $p \in [\bar{p}, 1)$, any SSPE of $G(p)$ is a no-delay SSPE.*

(ii) Suppose that Assumption 2(i) holds. Suppose that Condition 1 holds. Then, for any $p \in [0, 1)$, any SSPE of $G(p)$ is a no-delay SSPE.

(iii) Suppose that Assumption 2(ii) holds. Then, for any $p \in [0, 1)$, any SSPE of $G(p)$ is a no-delay SSPE.

The sketch of the proof is as follows. Suppose that p is sufficiently large. Let s be an SSPE of $G(p)$. For any state A, let u^A be the payoff tuple by s in any subgame of $G(p)$ starting with state A. We use induction. Let A be a nonsingleton state. Suppose that s involves no delay in any subgame of $G(p)$ starting with any state $B \subsetneq A$. Then, it suffices to show that in any round with state A, any proposer i's proposal by s is not rejected by any responder j. By Condition 1, $\left(b_i^{A\setminus\{j\}}, d_{-i}^A\right) \in \operatorname{int}\left(V^A(A, \{A\})\right)$. Note that by the induction hypothesis, as in Kawamori and Miyakawa (2016), $u_i^{A\setminus\{j\}} \approx b_i^{A\setminus\{j\}}$. Then, $\left(u_i^{A\setminus\{j\}}, d_{-i}^A\right) \in \operatorname{int}\left(V^A(A, \{A\})\right)$. Note that by convexity and strict superadditivity, $u^A \in V^A(A, \{A\})$, and $V^A(A, \{A\})$ is convex. Then, $x := pu^A + (1-p)\left(u_i^{A\setminus\{j\}}, d_{-i}^A\right) \in \operatorname{int}\left(V^A(A, \{A\})\right)$. Thus, there exists $y \in V^A(A, \{A\})$ such that $y \gg x$. By the induction hypothesis, any responder k's expected payoff by rejection is $pu_k^A + (1-p)\, d_k^A = x_k < y_k$. Thus, proposer i's proposal y is accepted. Hence, proposer i's payoff by proposal y is y_i. By the induction hypothesis, proposer i's expected payoff by any proposal rejected by responder j is $pu_i^A + (1-p)\,u_i^{A\setminus\{j\}} = x_i < y_i$. Thus, proposer i does not propose any proposal rejected by responder j.

5 Discussion

Based on a nontransferable-utility partition function form game, we defined a propose-respond bargaining game in extensive form, where the rejecter of a proposal exits from the game with a positive probability. We presented an example in which delay occurs. We provided sufficient conditions for no delay. Because one of the sufficient conditions for no delay is no externality and the example of delay is a nontransferable-utility one, externalities are necessary for delay, but nontransferable utilities are not.

It is an open question whether delay may occur in equilibrium under other type of partial breakdowns and externalities. Hart and Mas-Colell (1996), Miyakawa (2008) and Calvo (2008) considered partial breakdowns: in Hart and Mas-Colell (1996), the proposer exits from the game; in Miyakawa (2008), a responder is randomly selected and exits from the game; in Calvo (2008), a player is randomly selected and exits from the game. These studies showed that delay does not occur in any equilibrium. However, they did not consider externalities. Thus, it is unknown whether delay occurs when externalities are incorporated into their models.

Appendix

For any state A, any $i \in A$, any $p \in [0,1)$ and any stationary strategy tuple s of $G(p)$, let $u_i^A(s,p)$ be player i's expected payoff by s in any subgame of $G(p)$ starting with state A, and $u^A(s,p) := \left(u_j^A(s,p)\right)_{j \in A}$.

A Proof of Proposition 1

Let $p \in [0,1)$. Let $\hat{u}_1^N = \frac{1}{12}$, $\hat{u}_2^N = \frac{42-13p}{30(3-p)}$, $\hat{u}_3^N = \frac{75-23p}{60(3-p)}$, $\hat{x}_1^N = \frac{p}{12}$, $\hat{x}_2^N = \frac{-p^2-6p+36}{30(3-p)}$ and $\hat{x}_3^N = \frac{p^2-21p+72}{60(3-p)}$. Note that for any $i \in N$, $\hat{x}_i^N = p\hat{u}_i^N + (1-p)\, d_i^N$. Let s be the strategy tuple of $G(p)$, defined as follows.

- In any round with state N,
  - proposer 1 offers nothing to player 2 and $\hat{x}_3^N$ to player 3;
  - for any $i \in \{2,3\}$ and any $j \in N \setminus \{i\}$, proposer i offers $\hat{x}_j^N$ to player j;
  - for any $i \in N$ and any proposal x, responder i accepts x if and only if
    * any follower in the responding order accepts x and $x_i \geq \hat{x}_i^N$, or
    * some follower in the responding order rejects x and $p\hat{u}_i^N + (1-p) \cdot \frac{1}{4} \geq \hat{x}_i^N$.
- In any round with state A with $|A| = 2$,
  - for any $i \in A$, proposer i offers $\frac{p}{4}$ to the other active player;
  - for any $i \in A$ and any proposal x, responder i accepts x if and only if $x_i \geq \frac{p}{4}$.

s is obviously stationary.

Consider any round with state A with $|A| = 2$. Then, any proposal by s is accepted in s. Thus, any player's expected payoff by s is $\frac{1}{2}\left(\frac{1}{2} - \frac{p}{4}\right) + \frac{1}{2} \cdot \frac{p}{4} = \frac{1}{4}$. Hence, given the other actions in s, any responder's payoff by rejection is $p \cdot \frac{1}{4} + (1-p) \cdot 0 = \frac{p}{4}$. Hence, any response in s is optimal. Any proposer's payoff by s is $\frac{2-p}{4}$. Her payoff by any one-shot deviation to any proposal to be accepted in s is less than or equal to $\frac{1}{2} - \frac{1}{4}p = \frac{2-p}{4}$. Her payoff by any one-shot deviation to any proposal to be rejected in s is $p \cdot \frac{1}{4} + (1-p) \cdot 0 = \frac{p}{4}$. Note that $\frac{2-p}{4} - \frac{p}{4} = \frac{1-p}{2} \geq 0$. Thus, his/her proposal in s is optimal.

Consider any round with state N. Then, player 1's proposal by s is rejected by player 2 in s, and the other players' proposals by s are accepted in s. Thus,

$$
\begin{aligned}
u_1^N(s,p) &= \frac{1}{3}\left(pu_1^N(s,p)+(1-p)\cdot\frac{1}{4}\right)+\frac{1}{3}\cdot\frac{p}{12}+\frac{1}{3}\cdot\frac{p}{12}\\
u_2^N(s,p) &= \frac{1}{3}\left(pu_2^N(s,p)+(1-p)\cdot\frac{2}{5}\right)+\frac{1}{3}\left(1-\frac{p}{12}-\frac{p^2-21p+72}{60(3-p)}\right)+\frac{1}{3}\cdot\frac{-p^2-6p+36}{30(3-p)}\\
u_3^N(s,p) &= \frac{1}{3}\left(pu_3^N(s,p)+(1-p)\cdot\frac{1}{4}\right)+\frac{1}{3}\cdot\frac{p^2-21p+72}{60(3-p)}+\frac{1}{3}\left(1-\frac{p}{12}-\frac{-p^2-6p+36}{30(3-p)}\right)
\end{aligned}
$$

(on the right-hand side of each equation, the first (second, third) term corresponds to the payoff when player 1 (2, 3) is a proposer). Hence, $u_1^N(s,p)=\frac{1}{12}=\hat{u}_1^N$, $u_2^N(s,p)=\frac{42-13p}{30(3-p)}=\hat{u}_2^N$, and $u_3^N(s,p)=\frac{75-23p}{60(3-p)}=\hat{u}_3^N$. For any $i\in N$ and any proposal x, given the other actions in s, responder i's payoff by accepting x is x_i if any follower in the responding order accepts x, and $pu_i^N(s,p)+(1-p)\cdot\frac{1}{4}=p\hat{u}_i^N+(1-p)\cdot\frac{1}{4}$ otherwise; his/her payoff by rejecting x is $pu_i^N(s,p)+(1-p)\cdot\frac{1}{4}=p\hat{u}_i^N+(1-p)\cdot\frac{1}{4}=\hat{x}_i^N$. Thus, any response in s is optimal. Proposer 1's payoff by s is $p\cdot\frac{1}{12}+(1-p)\cdot\frac{1}{4}=\frac{3-2p}{12}$. Proposer 1's payoff by any one-shot deviation to any proposal to be accepted in s is less than or equal to $1-\frac{-p^2-6p+36}{30(3-p)}-\frac{p^2-21p+72}{60(3-p)}=\frac{3-2p}{12}-\frac{9(1-p)^2}{60(3-p)}$. Proposer 1's payoff by any one-shot deviation to any proposal to be rejected by responder 3 in s is $p\cdot\frac{1}{12}+(1-p)\cdot\frac{1}{4}=\frac{3-2p}{12}$. Thus, player 1's proposal is optimal. Proposer 2's payoff by s is $1-\frac{1}{12}p-\frac{p^2-21p+72}{60(3-p)}=\frac{2p^2-27p+54}{30(3-p)}$. Proposer 2's payoff by any one-shot deviation to any proposal to be accepted in s is less than or equal to $1-\frac{p}{12}-\frac{p^2-21p+72}{60(3-p)}=\frac{2p^2-27p+54}{30(3-p)}$. Proposer 2's payoff by any one-shot deviation to any proposal to be rejected in s is $p\frac{42-13p}{30(3-p)}+(1-p)\cdot\frac{1}{4}=\frac{2p^2-27p+54}{30(3-p)}-\frac{(1-p)(21-5p)}{20(3-p)}$. Thus, player 2's proposal is optimal. Proposer 3's payoff by s is $1-\frac{p}{12}-\frac{-p^2-6p+36}{30(3-p)}=\frac{7p^2-63p+108}{60(3-p)}$. Proposer 3's payoff by any one-shot deviation to any proposal to be accepted in s is less than or equal to $1-\frac{p}{12}-\frac{-p^2-6p+36}{30(3-p)}=\frac{7p^2-63p+108}{60(3-p)}$. Proposer 3's payoff by any one-shot deviation to any proposal to be rejected in s is $p\frac{75-23p}{60(3-p)}+(1-p)\cdot\frac{1}{4}=\frac{7p^2-63p+108}{60(3-p)}-\frac{3(1-p)(21-5p)}{60(3-p)}$. Thus, player 3's proposal is optimal.

Hence, s is a subgame perfect equilibrium.

Therefore, s is an SSPE that involves delay. □

B Proof of Proposition 2

Lemmas Let $p \in [0,1)$, s be an SSPE of $G(p)$ and A be a state. Suppose that for any state $B \subsetneq A$, s does not involve delay in any subgame of $G(p)$ starting with state B.

Lemma 1. $u^A(s,p) \in V^A(A, \{A\})$.

Proof. In the case where $|A| = 1$, the proposal x by s is *vacuously* accepted in s, and thus, $u^A(s,p) = x \in V^A(A, \{A\})$. Consider the other case. Let S be the set of players whose proposals are accepted in s. For any $i \in S$, let x^i be player i's proposal by s. Let $T := A \setminus S$. For any $i \in T$, let j_i be the first rejecter of player i's proposal in s. Note that for any $i \in T$, s does not involve delay in any subgame of $G(p)$ starting with state $A \setminus \{j_i\}$. Then,

$$u^A(s,p) = \sum_{i \in S} r_i^A x^i + \sum_{i \in T} r_i^A \left(p u^A(s,p) + (1-p) \left(u^{A \setminus \{j_i\}}(s,p), d_{j_i}^A \right) \right),$$

where $+$ is vector addition and $\sum$ is vector summation. Thus, because $p < 1$,

$$u^A(s,p) = \frac{\sum_{i \in S} r_i^A x^i + \sum_{i \in T} r_i^A (1-p) \left(u^{A \setminus \{j_i\}}(s,p), d_{j_i}^A \right)}{1 - p \sum_{i \in T} r_i^A}.$$

Note the following.

(a) For any $i \in S$, $x^i \in V^A(A, \{A\})$.

(b) For any $i \in T$, any player's proposal by s in any round with state $A \setminus \{j_i\}$ is in $V^{A \setminus \{j_i\}}(A \setminus \{j_i\}, \{A \setminus \{j_i\}\})$; it is accepted in s; thus, by convexity,

$$u^{A \setminus \{j_i\}}(s,p) \in V^{A \setminus \{j_i\}}(A \setminus \{j_i\}, \{A \setminus \{j_i\}\}) = V^A \left(A \setminus \{j_i\}, \pi_{j_i}^A \right);$$

hence, by strict superadditivity,

$$\left(u^{A \setminus \{j_i\}}(s,p), d_{j_i}^A \right) \in V^A \left(A \setminus \{j_i\}, \pi_{j_i}^A \right) \times V^A \left(\{j_i\}, \pi_{j_i}^A \right) \subset V^A(A, \{A\}).$$

(c) $\frac{\sum_{i \in S} r_i^A + \sum_{i \in T} r_i^A (1-p)}{1 - p \sum_{i \in T} r_i^A} = 1$.

Then, by convexity, $u^A(s,p) \in V^A(A, \{A\})$. □

Lemma 2. *Let $i \in A$ and x be a proposal of player i in any round with state A. Then, in any round with state A, if for any $j \in A \setminus \{i\}$, $x_j > pu_j^A(s,p) + (1-p)\, d_j^A$, player i's proposal x is accepted in s; if for some $j \in A \setminus \{i\}$, $x_j < pu_j^A(s,p) + (1-p)\, d_j^A$, player i's proposal x is rejected in s.*

Proof. Consider any round with state A. Suppose that for any $j \in A \setminus \{i\}$, $x_j > pu_j^A(s,p) + (1-p)\, d_j^A$. Let j be a responder. Suppose that any follower in the responding order accepts x in s. Note that s does not involve delay in any subgame of $G(p)$ starting with state $A \setminus \{j\}$. Then, given the other actions in s, responder j obtains x_j by accepting x and $pu_j^A(s,p) + (1-p)\, d_j^A < x_j$ by rejecting it. Thus, he/she accepts it in s. Hence, by the mathematical induction, x is accepted in s.

Suppose that for some $j \in A \setminus \{i\}$, $x_j < pu_j^A(s,p) + (1-p)\, d_j^A$. Suppose that x is accepted in s. Player j's payoff by s at his/her node at which he/she responds to x is x_j. Note that s does not involve delay in any subgame of $G(p)$ starting with state $A \setminus \{j\}$. Then, her payoff by the one-shot deviation to rejection is $pu_j^A(s,p) + (1-p)\, d_j^A > x_j$. Thus, the payoff by s is less than that by the deviation. This is a contradiction. Thus, x is rejected in s. □

Lemma 3. *Suppose that s does not involve delay in any subgame of $G(p)$ starting with state A. Let $i \in A$ and x^i be player i's proposal by s in any round with state A. Then, $f^A(x^i) = 0$, and for any $j \in A \setminus \{i\}$, $x_j^i = pu_j^A(s,p) + (1-p)\, d_j^A$.*

Proof. Let $y^i \in \mathbb{R}^A$ such that $y_i^i = x_i^i$, and for any $j \in A \setminus \{i\}$ $y_j^i = pu_j^A(s,p) + (1-p)\, d_j^A$. Because s does not involve delay in any subgame of $G(p)$ starting with state A, by Lemma 2, for any $j \in A \setminus \{i\}$, $x_j^i \geq pu_j^A(s,p) + (1-p)\, d_j^A = y_j^i$. Thus, $x^i \geq y^i$. Moreover, $f^A(x^i) \leq 0$. Hence, $f^A(y^i) \leq 0$. Suppose that $f^A(y^i) < 0$. Then, there exists $z^i \in \mathbb{R}^A$ such that $f^A(z^i) \leq 0$ and $z^i \gg y^i$. Note that for any $j \in A \setminus \{i\}$, $z_j^i > y_j^i = pu_j^A(s,p) + (1-p)\, d_j^A$. Then, by Lemma 2, player i's proposal z^i in any round with state A is accepted in s. Thus, by the one-shot deviation to proposal z^i, player i's payoff at his/her proposing node in any round with state A increases from x_i^i to z_i^i, which is a contradiction. Hence, $f^A(y^i) = 0$. Therefore, because $x^i \geq y^i$, $f^A(x^i) \leq 0$ and $f^A(y^i) = 0$, $x^i = y^i$, which implies the conclusion. □

Lemma 4. *Suppose that* $|A| \geq 2$. *Suppose that for any distinct* $i, j \in A$, $x^{ij} \in \operatorname{int}\left(V^A(A,\{A\})\right)$, *where* $x_i^{ij} = u_i^{A\setminus\{j\}}(s,p)$ *and for any* $k \in A \setminus \{i\}$, $x_k^{ij} = d_k^A$. *Then, in any round with state* A, *any player's proposal by* s *is accepted in* s.

Proof. Consider any round with state A. Let $i \in A$. Suppose that player i's proposal by s is rejected by some player $j \in A \setminus \{i\}$ in s. Then, proposer i's payoff by s is $pu_i^A(s,p) + (1-p)\,u_i^{A\setminus\{j\}}(s,p)$. Let $y^{ij} := pu^A(s,p) + (1-p)\,x^{ij}$. Note that by Lemma 1, $u^A(s,p) \in V^A(A,\{A\})$ and $x^{ij} \in \operatorname{int}\left(V^A(A,\{A\})\right)$. Then, by convexity, $y^{ij} \in V^A(A,\{A\})$. If $y^{ij} \in \operatorname{bd}\left(V^A(A,\{A\})\right)$, because $u^A(s,p) \in V^A(A,\{A\})$ and $x^{ij} \in \operatorname{int}\left(V^A(A,\{A\})\right)$, by the supporting hyperplane theorem, there exists $(a,b) \in \mathbb{R}^A \setminus \{0\} \times \mathbb{R}$ such that $a \cdot y^{ij} = pa \cdot u^A(s,p) + (1-p)\,a \cdot x^{ij} = b$, $a \cdot u^A(s,p) \geq b$, and $a \cdot x^{ij} > b$, which is a contradiction because $p < 1$. Thus, $y^{ij} \in \operatorname{int}\left(V^A(A,\{A\})\right)$. Hence, there exists $z^{ij} \in V^A(A,\{A\})$ such that $z^{ij} \gg y^{ij}$. For any $k \in A\setminus\{i\}$, $z_k^{ij} > y_k^{ij} = pu_k^A(s,p) + (1-p)\,d_k^A$. Thus, by Lemma 2, player i's proposal z^{ij} is accepted in s. Hence, proposer i's payoff by the one-shot deviation to proposal z^{ij} is $z_i^{ij} > y_i^{ij} = pu_i^A(s,p) + (1-p)\,u_i^{A\setminus\{j\}}(s,p)$, which contradicts that s is an SSPE. □

Proof of (i)

Lemma 5. *Let* A *be a state. Then, for any* $\epsilon \in \mathbb{R}_{++}$, *for some* $\bar{p} \in [0,1)$, *for any* $p \in [\bar{p},1)$, *for any SSPE* s *of* $G(p)$ *that does not involve delay in any subgame of* $G(p)$ *starting with state* A, $\left|\left|u^A(s,p) - b^A\right|\right| < \epsilon$.

Proof. Suppose that there exists $\epsilon \in \mathbb{R}_{++}$ such that for any $\bar{p} \in [0,1)$, for some $p \in [\bar{p},1)$, for some SSPE s of $G(p)$ that does not involve delay in any subgame of $G(p)$ starting with state A, $\left|\left|u^A(s,p) - b^A\right|\right| \geq \epsilon$. Then, for any $m \in \mathbb{N}$, there exists (s^m, p^m) such that $p_m \in \left[1 - \frac{1}{m}, 1\right)$, s_m is an SSPE of $G(p^m)$ that dos not involve delay in any subgame of $G(p^m)$ starting with state A and $\left|\left|u^A(s^m,p^m) - b^A\right|\right| \geq \epsilon$. Note that $1 = \lim_{m\to\infty}\left(1 - \frac{1}{m}\right) \leq \lim_{m\to\infty} p^m \leq 1$, and thus, $\lim_{m\to\infty} p^m = 1$. For any $m \in \mathbb{N}$, let x^{im} be player i's proposal by s^m in any round with state A. For any $m \in \mathbb{N}$, by Lemma 1, $u^A(s^m,p^m) \in V^A(A,\{A\})$, and $u^A(s^m,p^m) \geq 0$; thus, $u^A(s^m,p^m) \in V_+^A(A,\{A\})$. Hence, by the boundedness of $V_+^A(A,\{A\})$, $\left(u^A(s^m,p^m)\right)_{m\in\mathbb{N}}$ is bounded. Therefore, by the Bolzano–Weierstrass theorem, there exists a subsequence $\left(\left(\left(y^{im}\right)_{i\in A}, t^m, q^m\right)\right)_{m\in\mathbb{N}}$ of $\left(\left(\left(x^{im}\right)_{i\in A}, s^m, p^m\right)\right)_{m\in\mathbb{N}}$ such that $\left(u\left(t^m, q^m\right)\right)_{m\in\mathbb{N}}$ is convergent. By Lemma 3, for any $m\in\mathbb{N}$ and any distinct $i,j\in A$,

$$
\begin{aligned}
f^A\left(y^{im}\right) &= 0 \\
y_i^{jm} &= q^m u_i^A\left(t^m, q^m\right) + \left(1-q^m\right) d_i^A \qquad (4) \\
u_i^A\left(t^m, q^m\right) &= r_i^A y_i^{im} + \left(1-r_i^A\right) y_i^{jm}. \qquad (5)
\end{aligned}
$$

Thus, by Taylor's theorem, for any $m\in\mathbb{N}$ and any distinct $i,j\in A$,

$$
\begin{aligned}
0 &= f^A\left(y^{im}\right) - f^A\left(y^{jm}\right) \\
&= \left(y_i^{im} - y_i^{jm}\right)\frac{\partial f^A}{\partial x_i}\left(\theta y^{im} + (1-\theta)\, y^{jm}\right) - \left(y_j^{jm} - y_j^{im}\right)\frac{\partial f^A}{\partial x_j}\left(\theta y^{im} + (1-\theta)\, y^{jm}\right) \\
&= \left(1-q^m\right)\Bigg(\frac{1}{r_i^A}\left(u_i^A\left(t^m, q^m\right) - d_i^A\right)\frac{\partial f^A}{\partial x_i}\left(\theta y^{im} + (1-\theta)\, y^{jm}\right) \\
&\qquad - \frac{1}{r_j^A}\left(u_j^A\left(t^m, q^m\right) - d_j^A\right)\frac{\partial f^A}{\partial x_j}\left(\theta y^{im} + (1-\theta)\, y^{jm}\right)\Bigg).
\end{aligned}
$$

Note that because $\left(u\left(t^m, q^m\right)\right)_{m\in\mathbb{N}}$ is convergent, by (4) and (5), for any distinct $i,j\in A$,

$$
\lim_{m\to\infty} y^{im} = \lim_{m\to\infty} y^{jm} = \lim_{m\to\infty} u^A\left(t^m, q^m\right).
$$

Then, $\lim_{m\to\infty} u\left(t^m, q^m\right)$ satisfies (1) and (2). Thus, by Condition 1 ($d^A \in \operatorname{int}\left(V^A\left(A,\{A\}\right)\right)$), $\lim_{m\to\infty} u^A\left(t^m, q^m\right) = b^A$, which contradicts that $\left|\left|u^A\left(t^m, q^m\right) - b^A\right|\right| \geq \epsilon$ for any $m\in\mathbb{N}$. □

By Condition 1 and Lemma 5, for any nonsingleton state A, any $i\in A$ and any $j\in A\backslash\{i\}$, there exists $\bar{p}^{Aij}\in[0,1)$ such that for any $p\in\left[\bar{p}^{Aij},1\right)$ and any SSPE s of $G(p)$ that does not involve delay in any subgame of $G(p)$ starting with state $A\backslash\{j\}$, $\left(u_i^{A\backslash\{j\}}(s,p), d_{-i}^A\right)\in\operatorname{int}\left(V^A\left(A,\{A\}\right)\right)$. Because $\left\{\bar{p}^{Aij} \mid A\in 2^N \wedge |A|\geq 2 \wedge (i,j)\in A^2 \wedge i\neq j\right\}$ is finite, there exists a maximum $\bar{p}$ of this set, and $\bar{p}$ is in $[0,1)$. Let $p\in[\bar{p},1)$. Let s be an SSPE of $G(p)$. Let A be a state. Suppose that for any state $B\subsetneq A$, s does not involve delay in any subgame of $G(p)$ starting with state B. If $|A|=1$, *vacuously*, s does not involve delay in any subgame of $G(p)$ starting with state A. Consider the case where $|A| \geq 2$. Then, for any distinct $i, j \in A$, because $p \in [\bar{p}, 1) \subset [\bar{p}^{Aij}, 1)$, $\left(u_i^{A\setminus\{j\}}(s,p), d_{-i}^A\right) \in \text{int}\left(V^A(A, \{A\})\right)$. Thus, by Lemma 4, any player's proposal by s is accepted in s in any round with state A. Hence, s does not involve delay in any subgame starting with state A. Therefore, by the well-founded induction, for any state A, s does not involve delay in any subgame starting with state A. □

Proof of (ii)

Lemma 6. *Let A be a state. Then, for any $p \in [0,1)$, for any SSPE s of $G(p)$ that does not involve delay in any subgame of $G(p)$ starting with state A, $u^A(s,p) = b^A$.*

Proof. Let $p \in [0,1)$. Let s be an SSPE of $G(p)$ that does not involve delay in any subgame of $G(p)$ starting with state A. Let $i \in A$. Then, by Lemma 3,

$$u_i^A(s,p) = r_i^A\left(v^A(A,\{A\}) - \sum_{j\in A\setminus\{i\}}\left(pu_j^A(s,p) + (1-p)\,d_j^A\right)\right) + \left(1-r_i^A\right)\left(pu_i^A(s,p) + (1-p)\,d_i^A\right),$$

and thus,

$$(1-p)\,u_i^A(s,p) = r_i^A\left(v^A(A,\{A\}) - p\sum_{j\in A}u_j^A(s,p) - (1-p)\sum_{j\in A}d_j^A\right) + (1-p)\,d_i^A.$$

Note that by Lemma 3, $\sum_{j\in A}u_j^A(s,p) = v^A(A,\{A\})$. Then,

$$u_i^A(s,p) = r_i^A\left(v^A(A,\{A\}) - \sum_{j\in A}d_j^A\right) + d_i^A.$$

Therefore, by (3), $u_i^A(s,p) = b_i^A$. □

Let $p \in [0,1)$. Let s be an SSPE of $G(p)$. Let A be a state. Suppose that for any state $B \subsetneq A$, s does not involve delay in any subgame of $G(p)$ starting with state B. If $|A| = 1$, *vacuously*, s does not involve delay in any subgame of $G(p)$ starting with state A. Consider the case where $|A| \geq 2$. Then, for any distinct $i, j \in A$, by

Lemma 6, $b^{A\setminus\{j\}} = u^{A\setminus\{j\}}(s,p)$; thus, by Condition 1, $u_i^{A\setminus\{j\}}(s,p) + \sum_{k\in A\setminus\{i\}} d_k^A < v^A(A,\{A\})$. Thus, by Lemma 4, any player's proposal by s is accepted in s in any round with state A. Hence, s does not involve delay in any subgame starting with state A. Therefore, by the well-founded induction, for any state A, s does not involve delay in any subgame starting with state A. □

Proof of (iii)

Lemma 7. *Let A be a state. Then, for any $p \in [0,1)$, for any SSPE s of $G(p)$ that does not involve delay in any subgame of $G(p)$ starting with state A, $u^A(s,p) \geq d^A$.*

Proof. Let $p \in [0,1)$. Let s be an SSPE of $G(p)$ that does not involve delay in any subgame of $G(p)$ starting with state A. If $|A| = 1$, for any $i \in A$, $u_i^A(s,p) = \max W(A) = d_i^A$, and thus, $u^A(s,p) \geq d^A$. Consider the case where $|A| \geq 2$. For any $i \in A$, let x^i be player i's proposal by s in any round with state A. Suppose that for some $i \in A$, $u_i^A(s,p) < d_i^A$. By Lemma 1, $u^A(s,p) \in W(A)$; by strict superadditivity, $d^A \in \prod_{k\in A} W(\{k\}) \subset W(A)$; thus, by convexity, $pu^A(s,p) + (1-p)d^A \in W(A)$. By Lemma 3, for any $j \in A\setminus\{i\}$, $x_i^j = pu_i^A(s,p) + (1-p)d_i^A$; thus, $x_i^j - u_i^A(s,p) = (1-p)\left(d_i^A - u_i^A(s,p)\right) > 0$; because s does not involve delay in any subgame of $G(p)$ starting with state A, $u_i^A(s,p) = \sum_{j\in N} r_j^A x_i^j$; thus, $x_i^i < u_i^A(s,p) < pu_i^A(s,p) + (1-p)d_i^A$. By Lemma 3, for any $j \in A\setminus\{i\}$, $x_j^i = pu_j^A(s,p) + (1-p)d_j^A$. Therefore, $x^i \in \operatorname{int}(W(A))$, i.e., $f^A\left(x^i\right) < 0$. By Lemma 3, $f^A\left(x^i\right) = 0$, which is a contradiction. □

Let $p \in [0,1)$. Let s be an SSPE of $G(p)$. Let A be a state. Suppose that for any state $B \subsetneq A$, s does not involve delay in any subgame of $G(p)$ starting with state B. If $|A| = 1$, *vacuously*, s does not involve delay in any subgame of $G(p)$ starting with state A. Consider the case where $|A| \geq 2$. Let $i, j \in A$ such that $i \neq j$. By Lemma 7, $\left(u_i^{A\setminus\{j\}}(s,p), d_{-i}^{A\setminus\{j\}}\right) \leq u^{A\setminus\{j\}}(s,p)$, and by Lemma 1, $u^{A\setminus\{j\}}(s,p) \in W(A\setminus\{j\})$. Thus, by comprehensiveness, $\left(u_i^{A\setminus\{j\}}(s,p), d_{-i}^{A\setminus\{j\}}\right) \in W(A\setminus\{j\})$. In addition, $d_j^A \in W(\{j\})$. Note that by strict superadditivity, $W(A\setminus\{j\}) \times W(\{j\}) \subset \operatorname{int}(W(A))$. Then, $\left(u_i^{A\setminus\{j\}}(s,p), d_{-i}^A\right) \in \operatorname{int}(W(A))$. Thus, by Lemma 4, any player's proposal by s is accepted in s in any round with state A. Therefore, by the well-founded induction, for any state A, s does not involve delay in any subgame starting with state A. $\square$

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